TERMINAL BALLISTICS

An Interim Report to the Chief of Engineers, United States Army by The Committee on Passive Protection Against Bombing..............

NATIONAL RESEARCH COUNCIL
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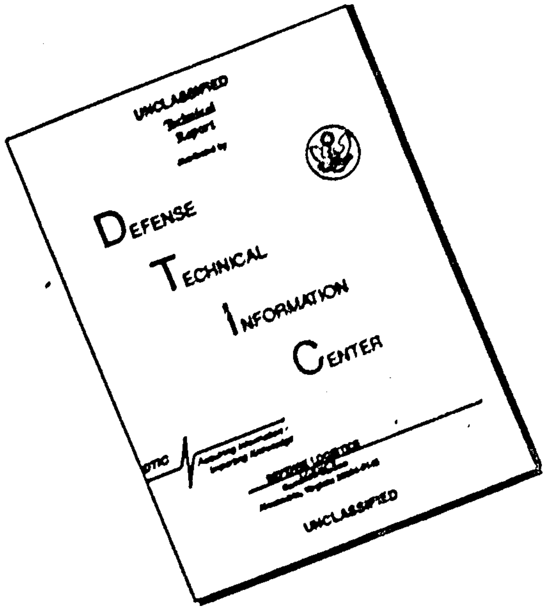

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NATIONAL RESEARCH COUNCIL

COMMITTEE ON PASSIVE PROTECTION AGAINST BOMBING

TERMINAL BALLISTICS

A Preliminary Report

by

H. P. Robertson
Theoretical Physicist for the Committee

WASHINGTON, D. C.........JANUARY 1941

This report, constituting a discussion of the theoretical and experimental results available at the present date, for the solution of the penetration problem, constitutes the first interim report rendered by the Committee on Passive Protection Against Bombing to the Chief of Engineers, United States Army, in accordance with terms of agreement between the United States of America and the National Academy of Sciences..............

Karl T. Compton..............
Luther P. Eisenhart..........
F. R. Moulton ....
Richard C. Tolman, Chairman..

* * *

John E. Burchard, Exec. Off..

The general problem confronting the Committee can be broken down into several subordinate problems, each of which may be attacked more or less independently of the others. This interim report deals with the present state of knowledge concerning one such aspect: namely, the ability of a given material, such as earth or concrete, to withstand the impact of a bomb, insofar as this ability is attributable to the localized properties, such as strength or density, of the material.

Part I deals with theories and data on the penetration of an inert bomb into an unlimited deformable solid or semisolid material. The depth of penetration may be expected to depend upon

a...properties of the bomb: weight W, caliber d, shape

b...striking conditions: velocity v_o, angle of incidence, yaw

c...properties of the target material: strength (such as compressive strength or hardness), density, porosity.

Since the latter elements (c) are ultimately responsible for the resistance R to penetration, it is most appropriate to base the investigation on the fundamental equation of motion

$$(1.1) \qquad \frac{W}{g} \frac{d^2x}{dt^2} = -R.$$

In all the theories considered R may be taken as the product $\varphi \cdot f$ of two factors, the first $\varphi(x)$ depending on the depth x of penetration already achieved, and the other $f(v)$ on the velocity v of the projectile at that stage. In the majority of the investigations found in the literature, this assumption is further specialized (Sec 2) by taking φ as a constant proportional to the cross-sectional area A of the bomb. The resistance may then be written

$$R = A\,f(v),$$

and the sectional resistance $f(v)$ may be interpreted as the energy necessary to displace unit volume of target material, by a projectile moving with velocity v ; in the units adopted in the report $f(v)$ is measured in lb/in^2, and one of the problems encountered is that of identifying f with some significant stress parameter of the medium. It is shown that such a resistance leads to a maximum depth of penetration x_1 of the form

$$x_1 = P\,F(v_0)\,, \tag{2.4}$$

where $P = W/A$ is the sectional pressure of the bomb, and $F(v_0)$ is a function of the striking velocity v_0 , which can be obtained by integration from the sectional resistance $f(v)$.

The principal sectional pressure formulae, associated with the names of Euler, Poncelet, and de Giorgi, are discussed in the remaining parts of Sec 2. Euler (Sec 2a) assumed the resistance to be a constant, whence

$$x_1 = \frac{P}{2\mu g} v_0^2 \quad ; \tag{2.7}$$

much of the recent German work (Peres) on air raid protection is based upon the use of this formulae. Values of the strength parameter $f = \mu$, for various substances and from various sources, are collected in Note 1 at the end of the report; most of them are, however, quite useless, because of inadequate data or specification.

The second formula considered (Sec 2b) is based upon the theory developed by Poncelet, supplemented by the admirable experimental program of the Metz Committee (1835). Poncelet's assumptions may be interpreted to mean that the projectile must not only supply energy to disrupt the cohesion of the material (as in the Euler hypothesis), but also energy to remove the detritus from its path; the resistance

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is then

$$R = A(a + b v^2) ,$$

where the "shatter strength" a is a measure of the cohesive force, corresponding to Euler's coefficient μ, and the constant b appearing in the inertial term is proportional to the density w' of the target material. The resulting penetration formula is

$$(2.10) \qquad x_1 = \frac{P}{2bgi} \log_e \left(1 + \frac{b}{a} v_0^2 \right) .$$

The Piobert-Morin-Didion values of a and b, for various media, are given in the original and in English units in Note 2, and the relation of b to the density of the medium is discussed in Note 3. Much of the modern work on penetration employs a special case of Poncelet's formula, due to Pétry (1910), in which b/a is given a definite numerical value, the same for all media; Pétry's formula has been transcribed here in the form

$$(2.13') \qquad x_1 = K P \log_{10} (1 + v_0^2/215000) ,$$

where the striking velocity v_0 is in ft/sec. Values of the parameter K, for various materials, are listed in Note 4.

The last of the sectional pressure formulae discussed in detail is that of Nobile de Giorgi (Sec 3c). De Giorgi reduced scattered penetration data in accordance with the general formula (2.4), and summarized the resulting $F(v_0)$ in tabular form; his F-values, reduced to standard English units, are given in Note 5.

The sectional pressure formulae of Euler, Poncelet, Pétry, and de Giorgi, for concrete or similar media, are compared graphically in Plates I and II - in Plate I on adjusting the strength parameters to bring the curves into agreement at around v_0 = 900 ft/sec, and in Plate II with the original parameter values. The application of the sectional pressure formulae of Euler, Pétry and de Giorgi to Aberdeen Proving Ground data on the penetration of projectiles and bombs into earth is given graphically in Plate IV.

In Sec 3 situations are considered in which the resistance encountered by the projectile may be expected to depend on the depth x of penetration. The first of these, discussed in Sec 3a, is the case of shallow penetration, in which the actual shape of the nose of the bomb may be expected to play a not inconsiderable role. The most natural modification of the expression for the resistance, traceable back to Morin (1836), leads to the penetration formula

$$(3.4) \qquad V(x_1) = W\,F(v_0)\,,$$

where $V(x_1)$ is the actual volume swept out by the projectile on penetrating to a depth x_1, and $F(v_0)$ is the velocity function previously introduced; this result is illustrated graphically in Plate III for the hypothetical case of a conical-nosed projectile piercing a thin plate.

The second non-sectional pressure theory considered is that of Vieser (1935), who professes to derive the penetration formula

$$(3.10) \qquad x_1 = \left[\frac{E_0}{2\sigma}\right]^{1/3}$$

from known elastostatic results, where E_0 is the striking energy of the bomb and σ is the compressive strength of the target material. Although there is little or no theoretical or empirical justification for this formula, Visser's many papers on the subject have had considerable influence on recent German literature on air raid protection, and it has therefore been thought desirable to include in Sec 3b an account and an attempted interpretation of his formula.

Part I concludes with a brief discussion of the deviations from the theoretical formula to be expected because of oblique incidence, yaw and the distortion of the bomb case.

In Part II rupture of the target, by spalling from the front face or scabbing from the back face, is discussed, with emphasis on the latter aspect. The formula

$$(5.4) \qquad s_1^3 = K_g W v^2 , \qquad (5.6) \qquad s_1^3 = K_h W v ,$$

proposed by de Giorgi and by Heidinger, respectively, for the scabbing depth s_1 caused by a projectile of weight W and velocity v are subjected to a critical examination. The formula

$$(5.7) \qquad s_1^2 = K_T W v^2$$

is proposed in the report, as an alternative which seems more plausible on theoretical grounds. But it is to be concluded that there is at present insufficient data available to allow a reliable treatment of scabbing.

Part III is devoted to the question of perforation of a target of given material and thickness e ; in Sec 6 to the case in which scabbing is inconsiderable, perforation being achieved by pure step-by-step penetration, and in Sec 7 to the case in which scabbing may play a more important role. In the former, application is made to the problem of determining the "limit velocity" v_1 of the plate - i.e., the velocity just sufficient to cause perforation. The formula

$$(6.8) \qquad v_1 = \sqrt{\frac{\pi a}{2}} \times \frac{d\, e^{\frac{1}{2}}}{m^{\frac{1}{2}}} \times (\text{fn of } \frac{e}{d})$$

is derived on the Poncelet hypothesis, and compared with various perforation formulae. The penetration cycle is discussed in Note 6, and the results illustrated graphically in Plate III. Sec 7 contains an exposition, following N. de Giorgi, of a method by which the combined effect of penetration plus scabbing may be determined from the separate penetration and scabbing formulae.

The final Part IV reviews briefly the conclusions which may be drawn from the investigation, with particular reference to the dependence of penetration on

a...weight and size of bomb
b...striking velocity
c...physical properties of the target.

(a) The data available in the literature seem to warrant the expectation that, for similar bombs, the depth of penetration (2.4) is proportional to the sectional pressure P of the bomb, with a coefficient $F(v_0)$ which depends on the striking velocity - i.e.

$$(2.4) \qquad x_1 = P\, F(v_0)$$

This implies a resistance

$$R = A\, f(v)$$

proportional to the cross-sectional area A of the bomb.

(b) Again, the available data seem to warrant the expectation that the sectional resistance $f(v)$ decreases as the velocity of the bomb decreases. It is considered significant that the empirical de Giorgi formula agrees closely with a formula of the Poncelet type (Plate I), and it is accordingly suggested that the assumption

$$(2.8) \qquad f(v) = a + b\,v^2, \qquad F(v) = \frac{1}{2bg}\log_e\left(1 + \frac{b}{a}v^2\right)$$

be entertained as a working hypothesis.

(c) The need for a more adequate correlation of elements entering into the penetration process with ascertainable physical properties of the target material is stressed. A theoretical argument favoring a sectional resistance $F(v)$ of the form (2.8) is advanced. In brief, it is argued that the actual mechanism of resistance involves both the overcoming of cohesion and the overcoming of the inertial reaction of the resulting detritus; it is contended that the two terms a and $b\,v^2$ entering into $f(v)$ above do, in fact, represent these two elements, and it is suggested that it may be profitable to determine to what extent they alone may be able to account for the observed phenomena. More specifically, and more tentatively, it is suggested that the former may be related to the hardness of the target material, and that the coefficient b in the latter may be represented (as in exterior ballistics) as the density of the material, modified by a suitable "drag coefficient." Notes 7 and 8 report ballistic and hardness tests which may be relevant to the determination of the "shatter strength" a.

The report concludes with a Glossary of symbols used, and a Bibliography of the principal references consulted in its preparation.

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INTRODUCTION

Before study can properly be made of the combination of materials in a structure, it is essential to gain some fairly clear concept of the forces to be resisted. In the problem under discussion, that of designing structures to resist the loads imposed by aerial bombardment, it is important to realize that these loads are dynamic and of magnitudes probably in excess of those dealt with in such efforts at dynamic structural design as have heretofore been made.

The first efforts of the Committee are, therefore, directed to obtaining a clearer conception of the forces which may be imposed on a structure by the falling and detonating bomb and it is therefore natural that the first studies should be of a physico-engineering problem, to wit "What size, thickness and physical properties of materials will be sufficient in the exposed members to absorb the immediate effects of impact and explosion?"

This question may provisionally be answered if solutions to a number of separate problems can be obtained. Among the more important of these would be

a...The resistance offered to penetration or perforation, insofar as this is due to the localized properties of the materials involved--e.g. the ability of an otherwise rigid slab to withstand a crushing blow.

b...The resistance offered to an explosion, detonated within or in the neighborhood of a given member, insofar as it is due to the localized properties of the materials involved.

c...The reaction of a member of given size and material, and having a given method of support, to impact or explosion, insofar as this is attributable to properties of the member *as a whole*-- e.g. the ability of a beam or column to withstand the sudden strains incident to the local resistance offered to impact and explosion.

The general purpose of the present report is the discussion of the theoretical and experimental results available, at the present time, for the solution of the penetration problem listed under (a) above. It will, in particular, deal with the penetration into and perforation through solid (concrete, steel, wood) and semi-solid (earth, sand) bodies. Because many of the penetration formulae found in the literature are based more or less directly upon a theoretical background, it is desirable to begin the study from a theoretical standpoint broad enough to include the more specific laws which have been proposed. The relation of these latter to the general theory will then be discussed, hand in hand with the experimental and observational evidence available in the literature, or otherwise made available to the Committee for this purpose. The report presents tentative conclusions based upon the present examination of the subject.

Princeton, New Jersey
November 4, 1940

H. P. Robertson

It may be tacitly assumed, for the present purposes, that the maximum demolition of an objective will be realized if the impact energy of the bomb or projectile is totally used up in penetration before the charge explodes; the study of penetration may then be separated from that of explosion.

Even if subsequent evidence should indicate that maximum effects do not result from this sequence, none the less the maximum penetration will be of interest.

It is to be expected that such maximum penetration of a bomb into a solid or semi-solid body will occur under the following conditions:

a...Incidence, normal to surface of body
b...Axis, tangent to trajectory
c...Motion, rectilinear
d...Case undistorted throughout penetration.

The study of penetration under these conditions, which are the most unfavorable from the standpoint of the defender, will lead to criteria of the most exacting requirements for design of shelters to resist penetration. Since it may, however, be within the power of the defender to alter certain of the above conditions by appropriate design, the effect of deviations from them will also be discussed.

It will be assumed throughout Part I, in discussing depth of maximum penetration, that the homogeneous target is thick enough to bring the bomb to rest; the problem of perforation, where the bomb breaks through the target, will be discussed in Part III.

1...General Theory of Penetration:

The resistance R offered by a body may be expected to depend on the depth of penetration x and on the velocity $v = dx/dt$ of the bomb at the moment t under consideration, as well as on certain parameters characteristic of the physical structure of the body (strength, density) and of the bomb itself (size, shape). See Fig 1*. The force of gravity may safely be ignored as utterly insignificant in comparison with the resistance R, at least so long as the bomb is actually in motion; the equation of motion of the bomb may, under these conditions, be taken as:

$$\frac{W}{g}\frac{dv}{dt} = -R(x, v)\,, \tag{1.1}$$

where W is the weight of the bomb, g the surface acceleration due to gravity, and R some function of the variables x and v**.

Now although R might be some quite general function of x and v, it is characteristic of the penetration theories here considered that in them the relative values of the resistance offered to bombs of various velocities v are the same for all penetration depths x; i.e. the resistance-velocity curves corresponding to any two depths are, to within a multiplicative factor, the same. This means that the resistance $R(x, v)$ is to be taken as the product of a function $\varphi(x)$ depending only on x, and a function $f(v)$ depending only on v.

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*To simplify the process of clear reproduction, and for purposes of easy reference, all Figures accompanying this report will be found grouped on pages at the end, so arranged that they can be opened out and read parallel with the text.

**English engineering units are adopted as standard for this report, although the corresponding Continental units may occasionally be used; conforming to military practice, W and R are measured in lb, x in ft, v in ft/sec, the caliber d of the projectile in in, and $g = 32.2$ ft/sec^2.

Since $\frac{dv}{dt} = \left[\frac{dx}{dt}\right]\left[\frac{dv}{dx}\right] = v\frac{dv}{dx}$, and

$R = \varphi(x) \cdot f(v)$, the equation of motion (1.1) may now be written

$$(1.2) \qquad \frac{W}{g} v \frac{dv}{dx} = - \varphi(x) f(v),$$

a form most suitable for integration. On introducing the two auxiliary functions

$$(1.3) \qquad \Phi(x) \equiv \int_0^x \varphi(x)\, dx, \qquad F(v) \equiv \frac{1}{g}\int_0^{v} \frac{v\, dv}{f(v)},$$

the integral of eq (1.2) for a bomb known to strike the surface $x = 0$ with the velocity v_0 may be written

$$(1.4) \qquad \Phi(x) = W\left[F(v_0) - F(v)\right];$$

from this the velocity v of the bomb at any depth x can be computed. If, in particular, the target is thick enough to bring the bomb to rest, then $F(v) = 0$, and the greatest depth x_1 to which it will penetrate is given implicitly by

$$(1.5) \qquad \Phi(x_1) = W F(v_0).$$

It will be found convenient, for many purposes, to represent the relation between velocity and depth of penetration graphically. Let the line OX in Fig 2 represent a line extending inward from the face of the target in the direction of motion, and let distances x from the face be measured along it. Take as ordinates the velocity v of the bomb, measured along OV; a point N a distance x from OV and distance v from OX then represents the instantaneous state of a bomb whose nose has penetrated to a distance x into the target and which has, at the depth, the velocity v. The point N_0 represents the bomb striking the face of the target with velocity v_0, and its velocity v at any subsequent position x will be represented by a point N on a curve N_0 N N_1 whose coordinates x, v satisfy eq (1.4); the point N_1 at which this curve intersects the X-axis, assuming sufficient thickness of target, then gives the depth x_1 at which the bomb comes to rest.

If it is desired to know the time required for penetration to depth x, eq (1.4) must be solved for v as a function of x and the resulting equation integrated, i.e.

$$(1.6) \qquad v \equiv \frac{dx}{dt} = G(x) \; ; \qquad t = \int_0^x \frac{dx}{G(x)} .$$

This quadrature requires, in all but the simplest cases, the application of numerical or graphical methods.

Further progress in the study of penetration can be made by either

a...Obtaining, from theoretical and other considerations, a more definite law for the resistance R, and hence for the functions Φ and F appearing in the penetration formulae.

b...Using the observational material on penetration depth as a function of the weight and striking velocity of the bomb to determine these functions empirically.

Both of these methods will be illustrated in the following survey of particular theories of penetration.

2...Sectional Pressure Theories. In General:

The majority of all theoretical or semi-empirical attempts to arrive at a specific form for the penetration formulae (1.4), (1.5), for depths x large compared with the caliber d of the projectile, are characterized by the assumption that $R(x, v)$ is

a...Proportional to the maximum cross-sectional area of the bomb, i.e.

$$A = \pi (d/2)^2 \tag{2.1}$$

b...Independent of the depth of penetration x.

It is then permissible to take

$$\varphi(x) = A\, i\,, \qquad \Phi(x) = A\, i\, x\,, \tag{2.2}$$

where i is a dimensionless constant which may depend on the shape of the bomb and which may, if desired, be absorbed in the other factor $f(v)$; the sectional resistance $f(v)$ depends above all on the physical properties of the target material, but may in addition be influenced by the shape of the bomb. This sectional resistance has the physical dimensions of pressure and is measured, in the units adopted as standard for this report, in lb/in^2 -- or, in the corresponding Continental system, in kg/cm^2.

The general penetration-velocity and maximum penetration formulae (1.4), (1.5) become under these assumptions

$$x = \frac{P}{i}\left[F(v_0) - F(v)\right], \tag{2.3}$$

$$x_1 = \frac{P}{i}\, F(v_0)\,, \tag{2.4}$$

where P is the sectional pressure of the bomb, i.e.

$$P \equiv \frac{W}{A} \tag{2.5}$$

It is measured, in the standard units, in lb/in^2. Thus, in any theory characterized by the two assumptions (a), (b) above, the depth of penetration of bombs of similar shape is directly proportional to the sectional pressure P. Such theories will, therefore, be referred to as sectional pressure theories; in them the weight and caliber of the bomb enter into the penetration formulae only in the combination P, although the form of the bomb may enter through the form factor i or otherwise.

The graphical representation discussed in Sec 1 assumes here a particularly simple form, for in the case of a sectional pressure theory, the velocity-distance curve for any striking velocity can be obtained from that for any other by sliding the curve as a whole along the X-axis. A glance at Fig 3 will illustrate this in a manner useful in the sequel, for on following the curve back from any maximum penetration depth x_1 the same curve is obtained in every case, except for its location along the line OX; read from this point of view, the curve is just that representing the relation (2.4) between striking velocity v_0 and maximum penetration x_1, as shown in the inset, Fig 3.

The more important sectional pressure theories found in the literature will be reviewed in the remainder of this section; relevant data will, for the most part, be relegated to the Notes at the end of the report. An attempt has been made to choose the notation in such a way as to avoid confusion on comparing the various formulae; this results in the multiplication of the number of symbols employed and, in many cases, in a deviation from the original notation. For these reasons, a Glossary of the symbols employed has been added at the end of the report.

2a...Robins-Euler:

A theory which plays an important role throughout the history of the subject is that in which the resistance R is assumed to be proportional to the maximum cross-sectional area A of the projectile and independent of the velocity v, as well as of the depth x. The relevant auxiliary functions are then given by (2.2) and

$$f(v) = \mu, \text{ const.}, \qquad F(v) = \frac{v^2}{2\mu g};$$

on setting the form factor $i = 1$, the equation of motion (1.2) may be written in the significant form

$$(2.6) \qquad dE = -\mu dV, \quad \text{where } E = \frac{W}{2g}v^2, \qquad dV = A\,dx$$

are respectively the kinetic energy of the bomb and the volume of target material displaced when the bomb moves an additional distance dx into the target -- at any stage in the motion after the bourrelet has penetrated the surface, as in Fig 4. This means that, neglecting the nose effect to be considered in Sec 3a below, the energy E required for penetration is proportional to the volume V displaced, and the constant of proportionality is interpretable as the energy required to remove unit volume of target material.* The maximum penetration x_1 and the time t_1 in which this penetration takes place are then

$$(2.7) \qquad x_1 = \frac{P}{2\mu g}v_0^2, \qquad t_1 = \frac{P}{\mu g}v_0$$

The second of these equations may be obtained by noting that the deceleration of the projectile is the constant $\mu g/P$, and, hence, the time t_1 is the quotient v_0/deceleration.

ROBINS

Perhaps the earliest quantitative investigation of penetration is that recounted by Benjamin Robins at the end of his *New Principles of Gunnery*, pub-

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*In the standard units μ is still in lb/in^2, for although E is in ft-lb, the volume V is in the rather unconventional unit $ft\text{-}in^2$ instead of ft^3 or in^3. A number of Continental writers, among them O. Spath and W. Vieser, have attempted to maintain a physical distinction between the units $cm\text{-}kg/cm^3$ and kg/cm^2.

lished in London in 1742*. In it, Robins states that the depth of penetration of a ball into a solid body is observed to be proportional to the square of its striking velocity, and concludes therefrom that the resistance is a constant. This result was elaborated by Leonard Euler in a note in his translation of Robins' treatise into German, published in Berlin in 1745**; starting from the constancy of the resistance, and assuming its proportionality to cross-sectional area, Euler deduces a formula for x_1 which is fully equivalent to (2.7) above. The data given by Euler lead to the values

EULER

$$\mu_{elm} = 22{,}000 \text{ lb/in}^2 , \quad \mu_{earth} = 1900 \text{ lb/in}^2$$

for elm wood and earth, in quite good agreement with more modern values.

This simplest theory recurs frequently in the literature of the subject; one of its most important rediscoveries from the present standpoint is that of W. Peres, published in the journal Gasschutz und Luftschutz in 1932.*** Peres computes the proportionality factor from published French and American data on bomb pen-

PERES

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*1742.1 - by which is meant the first item under the year 1742 in the Bibliography at the end of the report. Poncelet in 1838.1, p. 60, claims this distinction for his countryman the Abbé Camus, on the basis of an account "Sur l'action d'une balle de mousquet qui peut percer un corps solide sans le mouvoir sensiblement," Hist. acad. sciences 1738, pp. 98-101; but although Camus does give an account of the various factors entering into the problem, including mention of the resistance of the wood fibres, the results are qualitative rather than quantitative.

**1745.1, pp. 404 ff. of the Collected Works edition.

***1932.1, pp. 253-255.

etration; reduced to standard units, typical values of μ thus obtained are

$$\mu_{earth} = 2100 \text{ lb/in}^2, \quad \mu_{concrete} = 11{,}000\text{-}17{,}000 \text{ lb/in}^2$$

Values given for concrete by Heidinger*, obtained in part from the work of Nobile de Giorgi to be reviewed below, are higher, running from 14,000 lb/in^2 for poor or green concrete to as high as 49,000 lb/in^2 for reinforced concrete. These and other data on the Euler penetration formula are collected in Note 1 at the end of the report.

E. Gailer** calls this coefficient μ the "shatter strength" (Zertrümmerungsfestigkeit) of the material in the case of friable substances; he believes it to be some fraction of the "crushing strength" (Zermalmungsfestigkeit), which he reports Rosivel as finding to be some 1,400-28,000 (!) lb/in^2, with an average around 4,200 lb/in^2. Gailer claims the shatter strength has nothing to do with the compressive strength (Druckfestigkeit) of the substance, but suggests that it may be related to Schubfestigkeit (punching strength?).

2b...Poncelet:

The Robins-Euler formula is based on the assumption that the work required to displace unit volume of target material is independent of the velocity of the bomb, as well as of its depth. But it may be argued that, as in the case of a projectile moving through air, there should be an additional resistance depending on its velocity -- perhaps, as in the case of air, proportional to the square of the velocity. In the interpretation developed more fully below,

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*1936.2, p. 457.

**1937.2

the constant part of the resistance would represent the force required to overcome the cohesion of the target material, and the part depending on the velocity the force required to set the broken material into motion. Such a resistance was indeed postulated by J. V. Poncelet in the first edition of his Cours de mécanique industrielle, published in 1829. Since the first edition of this work is not available at the moment of writing this report, and since the author removed these considerations from the available second editions, the following account is based upon a non-mathematical discussion given by Poncelet in a report on the experimental work of two of his colleagues G. Piobert and A. Morin, and upon other sources*.

PONCELET

Let the velocity dependent factor $f(v)$ of R be taken as $a + b v^2$, where the coefficients a and b are to be considered as depending primarily on the target material, although the shape of the projectile may conceivably have an influence on them. The auxiliary functions φ and Φ are then given by eq (2.2), and from the above and eq (1.3) it follows that

$$(2.8) \qquad f(v) = a + b v^2 , \qquad F(v) = \frac{1}{2gb} \ln \left(1 + \frac{b}{a} v^2 \right),$$

(where ln stands for the natural logarithm to the base $e = 2.718....$). The depth of penetration x at which the velocity is reduced from v_0 to v is, by eq (2.3),

$$(2.9) \qquad x = \frac{P}{2gbi} \ln \frac{a + b v_0^2}{a + b v^2} ,$$

and by eq (2.4) the maximum penetration achieved by a bomb of striking velocity v_0 is

$$(2.10) \qquad x_1 = \frac{P}{2gbi} \ln \left(1 + \frac{b}{a} v_0^2\right) .$$

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*Poncelet 1835.1, pp. 57-78; Didion-Morin-Piobert 1836.1. See also Didion 1848.1, Sec. VII, and Cranz 1925.1, Vol. I, pp. 457-459. The notation adopted follows closely that of Cranz' more readily available Lehrbuch.

From eq (2.9), it can be shown that the time t_1 required to bring the bomb to rest is

$$t_1 = \frac{P}{gi\sqrt{ab}} \tan^{-1}\left(v_0 \sqrt{\frac{b}{a}}\right) . \tag{2.11}$$

The evaluation of the parameters a and b appearing in this theory is due largely to the experimental work of G. Piobert, A. Morin and Is. Didion, constituting the so-called "Metz Committee," in which balls were shot into various media, ranging from damp clay through various woods to limestone. The empirically determined values of these parameters are given both in the original units and in those standard for this report, in Note 2*. It is of interest in the present connection to remark that a great many of the reports on Air Raid Protection which have been published within the past few years, lean heavily on this century-old work for data on penetration; thus, all penetration data given in the Memorandum issued by the Institution of Civil Engineers in 1939** are obtained from the work here under review, with the exception of eight points -- and these latter are traceable to the 1911 papers of Nobile de Giorgi, to be reviewed in the sequel!

PIOBERT
MORIN
DIDION

Because of the historical importance of the Poncelet formula, and because this theory seems to offer the most promising background from which to begin the study of penetration, the mechanism of penetration to which it most natural-

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*From Didion 1848.1, pp. 241-244. Cranz 1925.1, p. 459 gives a fairly complete list.

**1939.3, pp. 42-43.

ly corresponds may be here elaborated.*

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"I assume that the work done by the bomb in moving a distance dx is expended in

a...Breaking the bonds which are responsible for the cohesion of solid or semi-solid material.

b...Supplying the resulting detritus with the kinetic energy required to get it out of the path of the bomb.

The first, I assume -- as in the Euler theory -- to be proportional to the volume $dV = A\,dx$ of material affected,

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*In order to emphasize naturalness of this interpretation, it may be pointed out that the entire description of the mechanism here given was arrived at by the writer quite independently of its historical roots, including the hypothesis that for shallow penetration, the resistance should be proportional to the amplitude of impression, as developed more fully in Sec 3a below. At that time, the only reference on hand was Cranz' Lehrbuch, containing a table of values of a and b/a; with the interpretation above in mind, the values of 2b were examined and found to be in good agreement with the densities of the materials. In some cases, the agreement was so close, it was suspected that the density may have been introduced as a part of the data; a search of the literature brought to light the references to Poncelet 1835.1 and Morin 1837.1, which revealed that the proportionality of b to the density ρ' was indeed contemplated at various times. But, relying on the explicit description in Didion 1848.1, p. 258, of the manner in which the coefficients b/a and subsequently b are to be obtained, it may still be believed that this hypothesis was not actually employed; if this conclusion proves to be valid, the good agreement obtained between 2b and ρ' in most cases constitutes a strong point in favor of the entire hypothesis. The original Piobert-Morin-Didion reduction (Mémorial d'artillerie No 4) is still not available to the Committee.

with a proportionality factor $\underline{a}$ which is independent of the velocity; this 'shatter strength' is a characteristic of the material, and is the subject of a more detailed investigation in Secs 3a and 8 below. Secondly, if it be assumed that the flow pattern of the crushed material is similar for different velocities v of the bomb (See Fig 5), then the average velocity v' imparted to this material is some multiple $\sqrt{\gamma}$ of v. The kinetic energy imparted to the debris dislodged during the displacement dx is then γ w' v^2 A dx/2g, where w' is the weight of unit volume of target material. The total resistance to the motion is therefore to be taken as

$$R = A\left(a + \gamma w' v^2/2g\right),$$

and on comparing this with Poncelet's equations, it is seen that

$$b = \frac{\gamma}{2}\frac{w'}{g} = \frac{1}{2}\gamma\rho', \tag{2.12}$$

where ρ' is the density w'/g of the medium. It can, at best, be hoped that the empirical values of b will be found proportional to $\frac{1}{2}\rho'$, with a factor of proportionality γ not too far from unity*; for a rough comparison, in the absence of quantitative information on the flow pattern, it should suffice for this purpose to set $\gamma = 1$. The results of such a comparison are exhibited in Note 3, in which $\frac{1}{2}\rho'$ is computed on the basis of available tables of specific gravity of substances of the kind investigated by the Metz Committee; not only is the order of magnitude, in general, correct, but also the relative values -- and, in many cases, the absolute values -- show a surprising agreement. I return to this point in the discussion of armor perforation in Sec 6 below.

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* $\pi\gamma/8$ will then correspond to the "drag coefficient" introduced in exterior ballistics; cf. Hayes 1938.1, p. 412.

I do not believe that the procedure adopted by Poncelet and Didion* for the determination of the form of the penetration cavity is well-founded -- and, according to Cranz, it leads to results which are in poor agreement with the observations. This procedure amounts to assuming that the volume of the crater down to any depth x is proportional to the kinetic energy expended by the projectile in reaching this depth, thus representing a retreat to the Euler hypothesis; it bears a resemblance, however, to the interpretation of Vicser's penetration formula offered in Sec 3b below." HPR

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PÉTRY

A modification of the Poncelet penetration formula, in a form involving only one parameter, was proposed by Pétry in 1910, presumably as the result of reworking of the old and the addition of new data. Pétry expresses the penetration depth in the form**

$$(2.13) \qquad x_1 = K_1 \frac{W}{d^2} f_n(v_o)$$

where $f_n(v_o)$ is given in tabular form. For the purposes of this report, it will be found more convenient to convert it into the form

$$(2.13') \qquad x_1 = K \; P \log_{10} (1 + v_o^2/215{,}000) \, ,$$

following the procedure of the handbook Civil Protection***

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*Didion 1848.1, p. 244; Cranz 1925.1, p. 463.

**1910.1, pp. 63-65.

***1939.2, pp. 140-141; mention is made here of the fact that Pétry's function $f_n(v)$ is in fact 10 $\log_{10} (1 + 50 \times 10^{-6} v^2)$ where v is measured in m/sec.

the velocity v_0 in eq (2.13') is to be measured in ft/sec. Values of K, corresponding to those given by Pétry and subsequent investigators using his formula, are given in Note 4.

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The work of Poncelet and contemporaries has been made the basis for developments by later writers; two such cases may be briefly mentioned as of possible interest in the present investigation.

In all of the above, it has been tacitly assumed that the projectile suffers no deformation on impact or during penetration. T. Levi-Civita* attempted to take such deformation on impact into account by replacing the area A by the larger <u>effective area</u>

$$(2.14) \qquad A' = A(1 + k_L v_0),$$

assuming that such deformation can be represented by a linear function of v_0. This additional factor could, if desired, be absorbed in the shape factor i.

H. Résal** proposed a resistance law for viscous semi-solids consisting of the sum of a term <u>linear</u> in the velocity and a quadratic term whose coefficient depends on the density in the manner described above; Résal found that, in order to fit certain of the data of Piobert and Morin, the coefficient corresponding to γ in the present treatment must be of the order 1/7. Résal's procedure offers an appropriate modification of the earlier French work for cases in which cohesion may be expected to give way to viscosity; in

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*1906.1.

**1895.1.

this connection, it may be noted that the decrease in relative importance of cohesion in comparison with inertia for semi-solids is shown by the rise of the value of b/a for masonry to 13 times this value for a mixture of sand and gravel.

2c...Nobile de Giorgi:

An important source of information on penetration, often called upon in recent work on Air Raid Protection, is to be found in the semi-empirical investigations of the Austrian Nobile de Giorgi*. By assuming that the resistance is independent of depth and that, as in air, it is proportional to the cross-sectional area of the projectile, de Giorgi deduces a general penetration formula equivalent to the general sectional pressure formula (2.4) above. In order to determine the function $F(v)$, de Giorgi eschews the resistance laws of Euler, Poncelet and Résal as only "simple analytical forms for approximate representation" and proceeds empirically on the basis of penetration data ranging from small arms up to 24 cm mortars. He finds that for groups of similar media (e.g. earth, gravel, sand), the form of $F(v)$ is the same, and that the penetration formulae for the various members of such a group can, therefore, be obtained by replacing $F(v)$ by $k\,F(v)$, where the parameter k characterizes one particular group member (e.g. $k = 0.45$ for broken stone, $k = 0.9$ for dry sand), and $F(v)$ is the same function of velocity for all members of the group. The penetration formula (2.4) may then be written

$$x_1 = \frac{P}{i} k\, F(v_0) , \tag{2.15}$$

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*1911.1, pp. 1003-1011, 1111-1123. For an excellent summary, in French, of the more relevant aspects of de Giorgi's work, see Montigny 1936.3.

where the factor i again depends on the shape of the projectile. The two principal groups of media considered by de Giorgi are (e) earth, sand, gravel, clay, etc., and (m) masonry, principally brick and concrete, and the velocity functions F_e, F_m characterizing these two groups are deduced from the data and given in tabular form. The values of i, k, F_e, and F_m obtained by de Giorgi are given, in the units adopted here, in Note 5.

The penetration values calculated from (2.15) should, according to de Giorgi, represent the average of a number of shots. He cautions that, because of inhomogeneities in the medium, rotation of the shell, etc., the maximum and minimum penetrations observed may, in the case of materials of the earth group, be 30% more or 25% less than the computed average, for striking velocities in the range 1000-1300 ft/sec.

No attempt is made by de Giorgi to discuss the dependence of resistance on velocity implied by the tabulated F values. It is, of course, possible to work backward from them to find the resistance; thus, for example, because of the approximately linear character of F_m between 650 and 1300 ft/sec, it follows immediately that in this range, the resistance is approximately proportional to the velocity of the bomb. In this same category is the empirical penetration formula

$$x_1 = \zeta P v_0 \tag{2.16}$$

due to Zalesky*.

Other sectional pressure theories are to be found in the literature, as for example, those of N. v. Wuich** and of Résal, mentioned above; they are, for the most part, mainly of

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*Quoted by Vieser 1934.2, p. 310.

**1893.1, 1st part.

historical interest, and consist chiefly in a reworking of the older data. In fact, a fairly thorough survey of the available sources makes it seem likely that the three theories discussed in detail in this section represent the principal contributions to the subject of penetration in materials other than armor plate. This belief is strengthened by the fact that most of the recent reports examined are based almost entirely on one or more of these three lines; thus the only penetration formulae contained in A R P Handbook No 5* and in the handbook Civil Protection** are, with one exception (Vieser), those of Poncelet, Pétry, and Peres, while, as mentioned before, the only ones given in the Institute of Civil Engineers' Memorandum are Poncelet and, via Bazant, points from de Giorgi. The situation in recent French, German, Czech, and Hungarian reviews seems to be about the same, with respect to sources of theories and data, as in English***.

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*1939.1, Ch. 2.

**1939.2, Ch. 13.

***Thus in the reviews of Montigny 1936.4; Bazant 1937.1, pp. 165-178; Vieser 1936.5, pp. 134-143; Heidinger 1936.2; Milota 1933.1, pp. 262-272; Harosy 1937.4, 1937.5.

3...Depth Effects:

Deviations from a sectional pressure formula for penetration are to be expected in any case in which the resistance depends on the depth of penetration. Thus, if the shatter strength were found to change with depth, such an effect, analogous to the barometer effect in exterior ballistics, would be expected; for practical purposes, however, it would probably be found preferable in such a case to consider the medium as compounded of homogeneous layers, in each of which the sectional pressure formulae would be applicable. The two cases which will be considered in some detail here are

a...That in which the length of the bomb is an appreciable fraction of the penetration depth, for then the actual shape of the nose may be expected to play a more complicated role than can be taken into account by the introduction of a semi-empirical form factor i .

b...That in which the shape of the crater produced indicates that the volume of material crushed per unit length of path increases with depth, as should be the case when large craters are produced whose external radius increases with increasing penetration, or when there is evidence of deformation of the bomb case.

The first of these effects may be expected to be of importance in dealing with exceedingly resistant targets, such as armor, and the second when dealing with a substance, such as concrete, under circumstances in which a considerable amount of spalling occurs; the two theories discussed below take these factors into account.

3a...Nose Effect. Morin:

That the resistance law during the initial stages of penetration should depend upon the actual area on contact with the target was recognized as long ago as 1835 by Arthur Morin, who included careful experiments on this effect in his classical observations on friction*. Morin's procedure consisted in dropping a ball from various heights and measuring the diameter of the impression produced on the surface of a bed of clay soil and of river sand; this work is of considerable historical interest in view of its close relationship to standard hardness tests, such as those of Brinell and Shore, in use today. The suggestion was again made by N. v. Wuich** in 1893, and yet again in 1936 by Heidinger***, who carried out penetration calculations for a conical-nosed bomb and applied them to the present problem of designing protection against aerial bombardment. And before these references were available, it occurred to the writer that such an effect was to be expected, and that it might play an important role in the theory of armor penetration. Although it was not felt, at the time, that such an effect would be of great significance for the present problem, a theory of it was developed along the following lines in the hope that on comparing its predictions with data on armor penetration, some understanding of the mechanism of penetration, in general, might be gained; the results of this and later comparisons are discussed in detail in Sec 6 below.

Let it, therefore, be assumed that the resistance encountered by the projectile is proportional to the area presented to the medium, projected onto a plane perpendicular to the direction of motion; in the case considered in this Part, this area is clearly the area of that part of the sur-

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*1835.1, Ch. III. Morin, in turn, quotes an older source of this hypothesis -- the Examen maritime of the Spaniard don Georges Juan, published originally in 1771 and in French translation in 1783.

**1893.1.

***1936.2, ¶ 3.

face disrupted up to the time under consideration, and has most appropriately been called the amplitude of impression*. It may be noted that the resistance would, indeed, be proportional to this area if the medium exerted a constant normal pressure on that part of the projectile in contact with it. During the first phase of the motion, before the bourrelet penetrates the surface, the amplitude of impression $A(x)$ when the nose is at depth x increases up to the maximum value A, eq (2.1), and maintains this value throughout the remainder of the motion -- or until the nose emerges from the rear face of the target, as considered in Sec 6 below. See Fig 6. If $d(x)$ denotes the diameter of the cross-section of the projectile at distance x from the nose, the amplitude of impression is

$$(3.1) \qquad A(x) = \pi\left[d(x)/2\right]^2 \quad \text{for } 0 \leq x \leq h$$
$$= A = \pi\left[d/2\right]^2 \quad \text{for } h \leq x ,$$

where h is the height of the ogive. The assumption upon which this section is based is then that the resistance to the motion is

$$(3.2) \qquad R(x, v) = A(x) \; f(v) ;$$

the auxiliary functions are given by eq (1.3) and

$$(3.3) \qquad \varphi(x) = A(x) , \quad \Phi(x) = \int_0^x A(x)\, dx = V(x) ,$$

where $V(x)$ is the volume swept out by the projectile in penetrating the target to depth x, and is called by Morin the volume of impression. The depth x at which the ve-

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*Attributed by Morin 1835.1, p. 709, to Georges Juan.

locity is reduced from the striking velocity v_0 to v. and the maximum depth x_1 are then given by

$$(3.4)\qquad V(x) = W\left[F(v_0) - F(v)\right], \qquad V(x_1) = W\,F(v_0),$$

cf. eqs (1.4), (1.5). The graphical representation of this result becomes almost as simple as in the case of sectional pressure theories if v be plotted against V, instead of against x, for then the curves for different striking velocities are the same, except for position along the V-axis.

The equation of motion for this case can be converted into the form

$$(3.2')\qquad dE = -f(v)\,dV(x), \text{ where } E = \frac{W}{2g}v^2;$$

this may be interpreted to mean that the energy necessary to displace unit volume of target material by a projectile of velocity v is given by the pressure $f(v)$. It may, for some purposes, be of value to compare the energy actually absorbed by the target with that required to push the projectile slowly through it to a depth x_1. The former is of course merely the striking energy E_0 and the latter is $f_0 V(x_1)$, where f_0 is the limit approached by $f(v)$ as v becomes negligible; note that f_0 is not the pressure at zero velocity, for that is zero -- or, strictly speaking, is equal to the sectional pressure $W/A(x_1)$, a quantity considered as negligible in comparison with the resistance throughout the motion. The ratio

$$(3.5)\qquad n = f_0\,V(x_1)/E_0 \;= f_0/\mu \quad \text{cf. (3.7) below}$$

may be taken as a measure of the efficiency of the penetration process; it is equal to unity in case the coefficient $f(v)$ is independent of velocity, and less than unity if $f(v)$ is an increasing function of velocity.

If the above comparison with the pressure f_0 required to push the bullet into the target is well-founded, then it should be possible to determine the value of f_0 for small velocities by one or another of the standard hardness tests, provided they are applicable to the type of material in question. Thus in the case of armor plate, this hypothesis would lead to the expectation that f_0 would be related to the Brinell hardness coefficient H of the plate; this relationship should be, at least to the first approximation

$$(3.6) \qquad f_0 = 100\ H\ \text{kg/cm}^2 = 1422\ H\ \text{lb/in}^2 ,$$

for the Brinell hardness coefficient is a measure of the pressure required, in kg/mm^2, to produce an impression of a steel ball on a metal surface. The possibility of employing a similar method for determining the shatter strength of friable materials is discussed in Sec 8 below.

Returning to the penetration problem, the velocity dependence of resistance must next be discussed; it may, in particular, follow any of the laws discussed in Sec 2. Thus if it be assumed independent of velocity, a simple modification of the Euler theory is obtained which leads to the penetration formulae

$$(3.7) \qquad V(x) = \frac{1}{\mu}(E_0 - E) \; ; \quad V(x_1) = E_0/\mu .$$

This procedure has, in fact, been followed by Heidinger in computing the penetration depths for a conical-nosed bomb*; as an illustration of the course of the resistance offered on this hypothesis, the computations for such a conical-

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*1936.2, pp. 455-461; 1937.3, pp. 269-274.

nosed projectile are given in some detail in Note 6.

The hypothesis which might be favored at the present time is that the resistance follows some law of the Poncelet type; this proposal was advenced as early as 1836 by A. Morin*, who announced that "the resistance is

a...Proportional to the area of a great circle of the projectile, or to the amplitude of the impression, and

b...A second factor consisting of two terms, one a constant depending only on the cohesion of the medium, and the other, due to the mobility of its molecules, proportional to the square of the velocity and to the density of the medium."

Under this hypothesis, the velocity at depth x and the maximum penetration x_1 are given by

$$(3.8)\quad V(x) = \frac{W}{2gb} \ln \frac{a + b v_0^2}{a + b v^2}, \quad V(x_1) = \frac{W}{2gb} \ln \left(1 + \frac{b}{a} v_0^2\right)$$

and the time at which the nose is at depth x by the integral

$$(3.9)\quad t = \int_0^x \frac{dx}{\sqrt{\varepsilon^{-2gbV(x)/W} (v_0^2 + a/b) - a/b}}$$

The course of the resistance and velocity curves for a conical-nosed bullet are illustrated in Note 6.

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*In a brief statement attributed to him in the report 1836.1 on a prize essay submitted by Didion, Morin, and Piobert.

The efficiency n , eq (3.5) , of the Poncelet process is

$$(3.5') \qquad n = \frac{a}{bv_o^2} \ln \left(1 + \frac{b}{a} v_o^2\right) .$$

3b...Crater Effect. Vieser:

A quasi-static theory of penetration into solid or semi-solid bodies has been developed and ardently promoted by the Austrian engineer W. Vieser in an extensive series of articles appearing during the past five years*. This theory is based upon Boussenesq's solution of the statical problem of determining the stress at any point in an isotropic, semi-infinite solid body obeying Hooke's law, due to a concentrated load W_s at a point on its surface. According to this solution, the stress at a point at depth x directly beneath the point of application of the load is

$$(3.9) \qquad \sigma_x = \frac{3 W_s}{2\pi x^2} .$$

Vieser now supposes that the bomb will penetrate to that depth x_1 at which this $\sigma_x = \sigma$, the compressive strength of the material, where W_s is the equivalent static load corresponding to a bomb of weight W and striking velocity v_o . In order to evaluate this latter, Vieser sets it equal to the average force E_o/x_1 required to stop a bomb of striking energy $E_o = W v_o^2/2g$ in a distance x_1 ; on substituting this value into eq (3.9), with $\sigma = \sigma_{x_1}$, and solving for x_1 Vieser finds, as the maximum

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*The principal articles are 1935.3, 1936.5.

depth to which such a bomb will penetrate

$$(3.10) \qquad x_1 = \left[\frac{3\,E_o}{2\pi\sigma}\right]^{1/3} \approx \left[\frac{E_o}{2\sigma}\right]^{1/3}$$

Similar results are obtained for loose material, such as sand or rubble, and expressed in terms of the general formula

$$(3.11) \qquad x_1 = \left[\frac{E_o}{\omega\sigma}\right]^{1/3}$$

where σ is the compressive strength, and ω is a dimensionless parameter whose value varies from 2/3 for loose, cohesionless media to 2 for elastic solids obeying Hooke's law.

Vieser's theory has been severely, but justly, criticized by Heidinger*, on the ground that the assumptions of classical elasticity theory are emphatically not fulfilled in the case here of interest, and although Vieser has retorted most acrimoniously to this criticism, there can be little doubt that his derivation is totally without theoretical justification, and, so far as can be seen from the evidence at hand, the resulting penetration formula is in flagrant discord with the facts.

Nevertheless, it is possible to arrive at an interpretation of Vieser's penetration formula (3.11) along the dynamical lines followed in this report. Presumably on Vieser's theory all material under a stress greater than the compressive strength σ would be crushed by the impact and subsequently blown out; hence the volume of material thus affected would be that contained within a surface at points of which the resultant stress is just equal to σ (Fig 7 a). But in Boussinesq's solution, the stress patterns for different loads W are similar, and hence the volume $V(x_1)$

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*1936.1; 1936.2, pp. 454-455.

of material removed in reaching the final depth x_1 is proportional to x_1^3. But on the modification of the Euler theory considered in the first part of the present section, the depth of penetration implied by (3.7) agrees, in its dependence on E_0, with Vieser's result. More specifically, if it were found that penetration resulted in the creation of a conical crater of constant semi-angle β, and that the detritus from various parts of the crater showed evidence of the same degree of crushing, this application of the Euler formula would yield

$$x_1 = \left[\frac{3E_0}{\pi \mu \tan^2 \beta}\right]^{1/3} \approx \left[\frac{E_0}{\mu \tan^2 \beta}\right]^{1/3}; \tag{3.12}$$

this is seen to be equivalent to Vieser's formula on setting $\omega \sigma = \mu \tan^2 \beta$.

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4...Oblique Incidence. Deviations:

If the projectile or bomb strikes the surface at an angle of impact Θ other than 0°, as in Fig 8, its subsequent motion depends upon a great many factors; above all, on the shape of the nose, rotation about its axis, and upon the nature of the surface struck. In the first place if, for a given projectile, the angle Θ exceeds some critical value Θ_r, the shell will ricochet rather than penetrate; according to the observations of Nobile de Giorgi*, Θ_r varies from 80° for loose earth to 40° - 60° for masonry or armor plate. This ricochet angle is influenced greatly by the shape of the nose, as well as by the nature of the surface, and by rotation and yaw of the projectile; a qualitative account of such influence is to be found in the work of de Giorgi and in Cranz**.

Assuming that conditions are such that penetration, rather than ricocheting, ensues, there are two principal procedures recommended for calculating the depth of penetration. The first, followed by de Giorgi in computing penetration at oblique incidence or for paths which deviate from rectilinearity, is simply to apply the normal rectilinear theory to measurement of distance along the actual path. Assuming no change of direction at impact, this would mean that depth of penetration x_1 would be computed from whichever formula is adopted by replacing x_1 by the length

$$\ell_1 = \frac{x_1}{\cos\Theta} \tag{4.1}$$

of the bore caused by the projectile, as illustrated in Fig 9; deviations from rectilinearity would lead to a more complicated relationship, which would, however, readily be obtained for each particular instance. The second method, used extensively in theories of armor perforation, is to re-

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*1911.1, pp. 1013-1015; reported also in Montigny 1936.3, pp. 397-399.

**1925.1, ¶ 78.

place the striking velocity v_o by its component

$$v_n = v_o \cos \Theta \tag{4.2}$$

normal to the surface. But according to de Giorgi, neither of these methods can be relied upon, and he therefore recommends the empirical procedure of setting

$$x_1 = \ell_1 \, (\cos \Theta)^{\nu} \tag{4.3}$$

and considering ν as a new parameter which is also to be evaluated from the data.

Deformation of the shell will also be expected to reduce penetration. The proposal of Levi-Civita, to multiply the maximum cross-sectional area by a factor depending linearly on the striking velocity and on the physical constitution of both the shell and the target, has been mentioned in Sec 2 above. Although there is no reason to believe that the effective area (2.14) can even be fairly well approximated by such a linear function of v_o, nevertheless, some such semi-empirical procedure might yield satisfactory results.

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The account of penetration in Part I above assumes that it proceeds gradually by changing the state of the target in the immediate neighborhood of the projectile, as well as that the target material is homogeneous and of sufficient thickness to stop the projectile. But it is a well-known fact that many substances, including concrete, may be perforated or otherwise damaged by an impulsive action which results in the formation of cracks or even in the detachment of large chunks of material. It is clear that in such cases, the energy required for the process must be transmitted through the material to points at some distance from the region directly attacked. An account of penetration would be incomplete if it did not include some reference to the phenomenon of <u>scabbing</u> or <u>spalling</u>, since this has, in many cases, been made an integral part of penetration studies. It may be discussed here, with the proviso that it may be found necessary to modify or revise the conclusions now reached, on taking into account the elastic properties of the target as a whole.

5...Scabbing:

Spalling occurs when relatively large masses of material are broken off from the target. In ordinary engineering, there has been little or no occasion to designate an effect of this sort in terms of the face on which it occurs. In dealing with problems of bombing, however, it is necessary to distinguish between the separation of material from the surface facing the bomb and the separation of material from the surface away from the bomb. Accordingly, whatever the etymological justification may be, in this report we shall follow the British terminology which uses:

a...Spalling -- to mean separation from the surface facing the bomb,

b...Scabbing -- to mean separation from the surface away from the point of impact of the bomb.

The theory and possible effects of these phenomena will be discussed here with emphasis on the case in which scabbing occurs. Spalling can then be treated by the same method, or in combination with those employed in Part I -- as, for example, in application to crater formation touched upon at the conclusion of Sec 3b. The reason for thus emphasizing scabbing is that it may play an important role in determining the ability of a projectile to perforate a protective member.

Scabbing may occur almost as soon as the bomb hits the target slab, as in the position (a) in the accompanying Fig 10 or after the bomb has penetrated some distance, as in (b); further, the plug or scab may even be ejected from a region which extends back to the nose of the bomb, as in (a) and (b), or it may be confined to a region at some distance from that directly in contact with the bomb, as in (c). In any case, there is a tendency for the plug to separate from the main body of the target along lines of diagonal tension and hence, in the materials under discussion, to be in the form of a right circular cone of apex angle 90°. And again, in any case, the energy required to overcome the cohesive forces must be gained at the expense of the kinetic energy of the bomb.

The forces effective in opposing scabbing are those operating along the sides of the cone, and it may be presumed that the total force is strictly proportional to the area of this latter. If the height of the plug be denoted by s the lateral area is proportional to s^2, and the total force S involved at any stage in the process may be taken as

$$(5.1) \qquad S = \tau s^2 ,$$

where τ is a measure of the relevant stress. If the rupture takes place during the time in which the bomb moves through a distance Δx, and in this interval the velocity of the bomb is reduced from v to v_r, then the energy lost by the bomb should for sufficiently small Δx be equal to that required to separate the plug, i.e.

$$(5.2) \qquad \frac{W}{2g}(v^2 - v_r^2) = \int_0^{\Delta x} \tau s^2 \, dx = U s^2$$

where U is the energy per unit area involved in the transaction. In particular, the maximum thickness s_1 which a bomb of weight W and velocity v will be able to break through is given by

$$(5.3) \qquad \frac{W}{2g} v^2 = U s_1^2 , \quad \text{where } U = \int_0^{\Delta x} \tau \, dx .$$

The treatment so far follows closely that of Nobile de Giorgi*, and seems to offer a justifiable basis on which to develop a method of calculating the greatest scabbing depth, in case the reaction on the bomb can, indeed, be considered as impulsive and not as due to the gradual accumulation of energy in the slab as might, for example, be the case in which a plug of type (c) were ejected in consequence of bending of the slab. But it is not clear just how to proceed in evaluating the integral U of the stress. In de Giorgi's original treatment, it is assumed that the distance Δx travelled by the bomb during the process is proportional to the thickness s of the remaining part of the slab, and that the mean value of τ is independent thereof; it then follows from eq (5.3) that the limiting thickness s_1 is defined by

$$(5.4) \qquad s_1^3 = K_g W v^2 ,$$

where the coefficient K_g depends only on the target material. The values of K_g given by de Giorgi are, reduced to the unit ft-sec^2/lb ,

for "very good brickwork" $K_g = 7.4 \times 10^{-7}$,

for "well-cured concrete" $K_g = 3.0 \times 10^{-7}$;

clearly here, as in Part I, such qualitative definitions as "very good" must be supplemented by quantitative data.

Heidinger** criticizes de Giorgi's treatment on the ground that:

> a...The manner in which the stress attains its ultimate value, and con-

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*1911.1, pp. 1007-1008.

**1937.3, pp. 266-269.

sequently the constant K, will depend on the form of the nose of the bomb, and

b...Although the derivation is based upon a dynamical foundation, the evaluation of the impulsive resistance is made along statical lines.

In his opinion, eq (5.4) will lead to too small values of s_1 for low velocities, and to too large values for high velocities. Heidinger proposes to take the influence of the time factor into account by returning to the original equation of motion and integrating with respect to t rather than x; eqs (5.2), (5.3), are then to be replaced by

$$(5.5)\qquad \frac{W}{g}(v - v_r) = \int_0^{\Delta t} \tau\, dt . s^2 , \qquad \frac{W}{g} v = \int_0^{\Delta t} \tau\, dt . s_1^2 .$$

Heidinger now proposes to adopt what he calls the "simplest assumption," that the time interval Δt in which the impulsive action takes place be proportional to s; in consequence of this assumption, he obtained in place of (5.4) a formula of the type

$$(5.6)\qquad s_1^3 = K_h W v$$

for the maximum thickness which can be broken through by a bomb of weight W and velocity v. The coefficient K_h is to depend on the shape of the bomb as well as on the target material; for a cylindrical-bodied bomb with nose fuze dropped against "high-quality" concrete, Heidinger gives a value which becomes, in the standard units,

$$K_h = 1.7 \times 10^{-4} \text{ ft}^2\text{-sec/lb} .$$

Other values recommended by Heidinger are to be found at the end of Note 5.

The derivations of both de Giorgi and Heidinger seem most unconvincing, and therefore an alternative hypothesis is proposed for the evaluation of maximum scabbing thickness. It may be assumed that the energy per unit area U, eq (5.3) required to overcome the cohesive forces is independent of the velocity of the process; this assumption represents the extension to the breaking point of a situation which does hold within the yield point and for low rates of strain.* Under this assumption, the maximum thickness s_1 is given by

$$(5.7) \qquad s_1^2 = K_r W_v^2 ,$$

i.e. it is proportional to the square root of the striking energy.

In order to represent the scabbing curve graphically, it is convenient to let 0 0' be the path of the bomb, where 0' is the point of emergence on the rear face of the target, and to measure distances s from the back of the target; again let the velocity v of the bomb be measured along an axis perpendicular to 0 0', as illustrated in Fig 11. A point N on this diagram will then represent the state of a bomb whose velocity is v and whose nose is at distance s from the rear face. The relation between s and v given by one of the scabbing formulae is then represented

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*This is known, however, to be only approximately correct for metals, even within the velocity range here under consideration, and may be only approximate for other materials. Thus, as reported by Manjoine and Nadai (at the meeting of the American Society for Testing Materials in Atlantic City on June 24-28, 1940), the true tensile strength of pure copper at room temperature rises some 22% over a range in which the rate of strain is increased by a factor of 10^6, and this must, in turn, have some effect on the energy required for rupture.

graphically by the locus of points N whose velocity and distance, s, are such that, under the most unfavorable conditions to the structure, scabbing will just ensue. Hence points in the region S lying above this curve represent states in which scabbing can occur, the bomb then emerging from the rear face with a residual velocity. Points in the region P below the curve represent states in which the bomb lacks enough energy to cause scabbing; whether in such a case scabbing can occur at a later stage of penetration will depend upon the relative positions of the penetration and scabbing curves, as will be discussed in Sec 7 below.

In order to compute maximum thickness of scabbing for impact at angle Θ, de Giorgi simply replaces v by its normal component $v \cos \Theta$ -- i.e. he assumes that the component of v parallel to the face of the target has no effect on the phenomenon.

In Part I, a study was made of the motion of a projectile in a resisting medium, principally solid or semi-solid, in which the resistance was due to the ultimate strength and inertia of those parts of the medium in proximity to the projectile; this penetration process is a gradual one, governed by a differential equation. Part II dealt, on the other hand, with cases in which the resistence was due to the transmission of disruptive forces to more distant regions and, under the assumptions there adopted, could be considered as an impulsive process resulting in the breaking off of a large mass of material within a relatively short time. Now the perforation of a given slab of material by a bomb may take place in accordence with either or both of these mechanisms; the study of the manner in which this may take place is the object of this Part.

6...Perforation Without Rupture. Armor Perforation:

Perforation may be accomplished by "pure" penetration, without scabbing; whether break-through occurs in this manner or not depends primarily on the nature of the target material. Such seems to be the case in many experiments and observations on the perforation of light armor plate, and the development in the present section will therefore be made with this possible application in mind. Suppose, then, that the plate is of thickness e and that the resistance law is that considered in Sec 3a, i.e. that the energy lost by the projectile in moving a distance dx is proportional to the volume $dV(x)$ displaced, with a coefficient $f(v)$ which may depend on the velocity v. For penetration depths $x \leq e$, the resistance R is again given by eq (3.2) above, and the velocity at this depth by the first of the eqs (3.4). But for $x > e$, where the projectile has already begun to emerge from the rear face of the plate, as in positions (b) and (c) in Fig 12, the amplitude of impression $A(x)$ employed in Sec 3a must be replaced by the projection of the part of the ogive actually confronted by plate material, and $V(x)$ by the volume of plate material actually displaced. This may readily be accomplished by replacing $A(x)$, $V(x)$ in these formulae by the amplitude and volume of the impression

$$(6.1) \qquad A(x, e) \equiv A(x) - A(x - e) ,$$

$$V(x, e) \equiv V(x) - V(x - e) ;$$

whence

$$(6.2) \qquad R(x, v) = A(x, e) f(v) ,$$

$$V(x, e) = W \left[F(v_0) - F(v)\right] .$$

These formulae (6.2) may now be considered as holding for all penetration depths x, provided $A(x-e)$, $V(x-e)$ be defined as zero for negative arguments $x - e$. As an illustrative example, the case of a conical-nosed projectile is considered in some detail in Note 6.

The limiting thickness of plate which can just be completely perforated by a projectile of striking velocity v_o can be obtained from the second of eqs (6.2) by setting $V(x, e)$ equal to the volume $\pi (d/2)^2 e$ of the cylindrical hole punched in the plate and solving for e on setting $v = 0$, whence

$$(6.3) \qquad e = \frac{4mg}{\pi d^2} F(v_o), \quad \text{where } m \equiv W/g$$

is the mass of the projectile. Conversely, the "limit velocity" v_o for a plate of thickness e is obtained by solving (6.3) for v_o; it is seen that for a given target material the limit velocity is, under the hypotheses here adopted, a function of $d^2 e / m$, and since for similar projectiles of the same mean density the mass varies as d^3, it follows that for them the limit velocity is a function of the ratio e/d.

For a more precise prediction of limit velocity, the dependence of resistance on velocity and on the physical properties of the plate must be specified; the two possibilities considered here are those associated with the names of Euler and of Poncelet in Sec 2 above. The former leads to a perforation formula which may be expressed in either of the two equivalent forms

$$(6.4) \qquad \frac{E_o}{A} = \mu e, \quad \text{or} \quad v_o = \sqrt{\frac{\pi \mu}{2}}\,\frac{d\, e^{\frac{1}{2}}}{m^{\frac{1}{2}}};$$

the first of these is, for $\mu = 10{,}000$ kg/cm^2 [corresponding under the hypothesis expressed by eq (3.6) to a Brinell hardness coefficient $H = 100$,] the exact expression of a rough rule formerly employed by the Krupp Works for wrought iron armor -- that "a projectile will penetrate as many decimeters as is given by its sectional energy measured in meter-tonnes per square centimeter."*

It is well established, however, that the limit velocity for armor plate increases with e at a rate faster than that predicted by eq (6.4), as witnessed by the classical formulae of de Brettes, de Marre, Weaver and others**, and above all by the more modern work of L. Thompson to be discussed below. Now such an effect is to be expected whenever $f(v)$ increases with velocity, for then $F(v)$ will show a slower increase with velocity than in the Euler case. This expectation is borne out on adopting the Poncelet hypothesis

$$(6.5) \qquad f(v) = a + \frac{\gamma}{2}\rho' v^2 , \quad F(v) = \frac{1}{\gamma\rho' g} \ln(1+\alpha v^2),$$

where γ is again some dimensionless parameter analogous to the drag coefficient for air, ρ' is the (mass) density of the target material, and $\alpha = \gamma\rho'/2a$. The second of eqs (6.2) assumes, on multiplying it by $\gamma\rho'$, the simple form

$$(6.6) \qquad m \ln\frac{1 + \alpha v_0^2}{1 + \alpha v^2} = \gamma m'(x) ,$$

where $m'(x) \equiv \rho' V(x,e)$ is the mass of target material displaced at projectile penetration x. Hence on complete perforation

$$(6.7) \qquad m \ln (1 + \alpha v_0^2) = \gamma m' ,$$

$$\text{or} \quad v_0 = \left[(\epsilon^{\gamma m'/m} - 1) /\alpha\right]^{\frac{1}{2}} ,$$

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*Quoted from N. v. Wuich 1893.1, 2nd part.

**See de Giorgi 1912.1, pp. 20-21; Cranz 1925.1, pp.464-465.

where $m' = \pi(d/2)^2 e \rho'$ is the total mass displaced. On replacing α by its value, given under (6.5) above, the limiting velocity may be written in the significant form

$$(6.8) \qquad v_0 = \left(\frac{\pi a}{2}\right)^{\frac{1}{2}} \frac{d\, e^{\frac{1}{2}}}{m^{\frac{1}{2}}} \bar{f}\left(\gamma \frac{m'}{m}\right) \quad ,$$

where

$$(6.9) \qquad \bar{f}(z) \equiv \sqrt{(\varepsilon^z - 1)/z} = 1 + \tfrac{1}{4} z + \frac{5}{96} z^2 \quad \ldots .$$

for small values of z .

The perforation formula (6.8) does show an increase with e of the kind expected on the qualitative argument advanced above, for m' increases directly with e . In order to examine this dependence more closely, let λd be the "reduced length" of the projectile, i.e. the length of a cylinder of the same caliber d , density ρ and total mass m as the actual projectile. Then

$$(6.10) \qquad \gamma \frac{m'}{m} = \gamma \frac{\pi (d/2)^2 e \rho'}{\pi (d/2)^2 \lambda d \rho} = \chi \frac{e}{d} \quad ,$$

where $\chi \equiv \gamma \rho' / \lambda \rho$; hence for similar projectiles the function $\bar{f}$ appearing in eq (6.8) should depend only on the ratio e/d . Such a theoretical formula has, in fact, been obtained by L. Thompson and F. B. Scott* by dimensional reasoning, and has been applied by Thompson to data ob-

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*1927.1; see also Thompson 1930.1, 1932.2 Had these authors assumed that the thickness e of the plate could enter only through the mass m' of target material ejected, they would have been led, as in the above, to a dependence on m'/m instead of on e/d.

tained at the U. S. Naval Proving Ground at Dahlgren, Va.; for the case of normal incidence, Thompson's formula may be written

$$(6.11)\qquad v_0 = \bar{k}\,\frac{d\,e^{\frac{1}{2}}}{m^{\frac{1}{2}}}\,\bar{F}\left(\frac{e}{d}\right) .$$

Comparison with eq (6.8) suggests that

$$(6.12)\quad \bar{k} = \left(\frac{\pi a}{2}\right)^{\frac{1}{2}} ,\quad \bar{F}\left(\frac{e}{d}\right) = \bar{f}\left(\chi\frac{e}{d}\right) = 1+\frac{\chi}{4}\frac{e}{d}+\dots.$$

provided Thompson's function $\bar{F}$ be normalized to have the value 1 for $e = 0$.

To test this hypothesis, the predicted values of $(\pi a/2)^{\frac{1}{2}}$, or a itself, and the initial slope $\chi/4$ of the $\bar{F}(e/d)$ curve should be compared with those found empirically. It is rather difficult, for obvious reasons, to find modern data in the literature with the aid of which these comparisons may be carried out; with the exception of a comparison with such of Thompson's semi-quantitative data as have been published, it is necessary to restrict this report to a comparison with older results.

The problem of determining a satisfactory empirical formula for the perforation of armor plate of wrought iron was attacked repeatedly between 1870 and 1884 by Martin de Brettes, who finally evolved the formula*

$$(6.13)\qquad T^m = 0.073\ e + 0.027\ e^2/d$$

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*1884.1.

for the sectional energy, in meter-tonnes per cm^2, necessary to penetrate a wrought iron plate of thickness e(cm) by a projectile of caliber d(cm). This leads to a limiting velocity

$$(6.14) \qquad v_o = \sqrt{\frac{73\pi}{2}}\,\frac{d\,e^{\frac{1}{2}}}{m^{\frac{1}{2}}}\,(1 + 0.370\ e/d)^{\frac{1}{2}}$$

$$= \sqrt{\frac{73\pi}{2}}\,\frac{d\,e^{\frac{1}{2}}}{m^{\frac{1}{2}}}\,(1 + 0.185\ e/d + \ldots) \,,$$

and to a value $a = 7{,}300\ kg/cm^2 = 104{,}000\ lb/in^2$. A similar formula attributed to Krupp* leads to $a = 7{,}600\ kg/cm^2$ and to a coefficient of e/d of 0.158 in place of de Brettes' 0.185. In order to compare these results with the theoretical prediction (6.12), one must know the equivalent length λ d of the projectiles used, and must make some estimate of the coefficient γ; it would seem safe to assume that the projectiles and the target are of about the same density. An examination of some of the data on projectiles quoted by de Brettes indicates an equivalent length of around 1.9 calibers, and it can be assumed that γ should not differ too greatly from unity. Under these assumptions, the theoretical coefficient of e/d in f turns out to be $0.132\,\gamma$, as compared with de Brettes' 0.185 and Krupp's 0.158. Further, the value of a is found to compare favorably with that which would be predicted on the hypothesis concerning its connection with hardness advanced in Sec 3a above, for according to (3.6), the coefficient a, in kg/cm^2, which here represents the resistance per unit area encountered at negligible velocities, should be just 100 times the Brinell hardness coefficient H. Although the writer cannot know the Brinell coefficient of the armor investigated by de Brettes, on using the value H = 85 found in the literature for "pure iron,"** the val-

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*See de Giorgi, 1912.1, p.21.

**Given by F. G. C. Batson in the October 1918 issue of Proc. Inst. Mech. Engineers, p. 593.

ue 8,500 kg/cm^2 is obtained, for comparison with de Brettes' 7,300 and Krupp's 7,600; this point will bear a closer examination than has been possible to give it at this time. It is to be noted that, as should be expected, the values of a on the Poncelet hypothesis are lower than the corresponding values of μ on the Euler hypothesis; this is also illustrated in the example worked out in Note 6 at the end of the report. Only a rough check can be attempted with Thompson's more recent observations on perforation of armor plate. The accompanying Fig 13 represents all the data given for normal incidence in Thompson's 1930 article, and although neither the abscissa nor ordinate scale is given there, it may be reasonable to assume that the lines framing the bottom left corner of the figure are the $\bar{F}$- and e-axes. Now from a passing remark in Thompson's 1932 article, the writer feels justified in assuming that the abscissa $e/d = 0.45$ is in some such position as that indicated in the figure, and therefore the triangle on this line is chosen somewhat arbitrarily as a fixed point through which to pass the theoretical curve. Examination of an illustration of an A. P. projectile in Hayes' Elements of Ordnance* indicates that it has a reduced length of about 3 calibers, and since the mean density of such a shell should not differ greatly from that of armor, again $\rho' = \rho$. Under these conditions, the theoretical curve for $\gamma = 1$ is that represented by the linear function

$$(6.15) \qquad \bar{F}\left(\frac{e}{d}\right) = 1 + \frac{1}{12}\frac{e}{d} \quad ,$$

to within 1% over the range of e/d in question; this theoretical line is that shown on the diagram. The agreement is as close as could be hoped for within a straight-line graph;

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*1938.1, p. 314.

it may be found possible to account for the deviations observed in thin plates in terms of bending, but in any case, the experimental points seem to show a lack of reproducibility in this region which could hardly be accounted for by the factors here taken into account.

In concluding this section, it may be remarked that the interpretation of the Poncelet hypothesis adopted in Part I leads to a plate perforation formula which is in at least fair agreement with the available data.

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7... Penetration plus Scabbing:

Perforation may be caused either by the penetration extending all the way through the thickness e of the target, as in the case considered in Sec 6 above, or by scabbing occurring at some phase in the process, either at the time the projectile hits the target or after some penetration. In any case, there may be a competition between the two processes of penetration and scabbing for the kinetic energy of the projectile; assuming there is enough energy available for scabbing, which course is actually followed may depend on the nature of the target material, and perhaps also on the shape of the nose of the projectile. It is tacitly assumed in the literature that in the case of a material in which scabbing is known to occur, such as concrete, the necessary energy will actually go into the process.* Certain it is that if this assumption is made, the calculated limiting thickness at which perforation will just occur will be the largest which can be obtained for a given striking velocity, and recommendations based upon such calculations will be on the safe side. It would, therefore seem advisable to assume, in the absence of more definite information, that scabbing will occur if and when there is energy enough available for it, at least in the case of friable materials, and the remainder of this section will be based upon this assumption.

The problem of whether a slab of thickness e can be broken through by a given projectile with a given striking velocity v_0 can now be solved graphically by a combination of the methods described in Secs 1 and 5 above. Let OO', Fig 14, represent the path of a bomb through the slab, and measure velocity perpendicular thereto. The scabbing equation for the material involved can then be laid off proceeding backwards from O', as in the figure; let it intersect the ordinate OV at the point S, as shown. If now the striking velocity v_0 of the projectile is greater

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*E.g. de Giorgi 1911.1, pp. 1122-1127; Heidinger 1937.3, pp. 270-274.

that represented by OS , as it is at the position labelled I , scabbing may occur -- and, in accordance with the considerations outlined above, it is here to be assumed that in such a case scabbing will occur. On the other hand, if v_0 is less than the scabbing velocity represented by OS, penetration will ensue. Now if, as indicated in the position II, the penetration curve drawn into the target intersects the scabbing curve, scabbing will be assumed to occur when the bomb has penetrated to this position; if not, as in III, the bomb will eventually come to rest within the target. The limiting position R at which the penetration curve just touches the scabbing curve defines the upper limit to the striking velocity which the given target thickness can withstand; the points below R on OV represent striking velocities which will not suffice to perforate the target. If this procedure be repeated for various thicknesses OO' , keeping the position of O' fixed and varying that of O , the entire x-v diagram can be divided into at most three regions: the region S in which scabbing may occur, a second region P in which only partial penetration can ensue, and a third region P + S , in which penetration may be followed by scabbing. This latter region P + S may, theoretically, be absent, for it may happen that the penetration curves are steeper at each point than the scabbing curve at the corresponding value of v . The curve R separating the region P of partial penetration from S and P + S , in either of which perforation may occur, may be called the rupture line; only those states represented by points lying below R are surely safe.

In the case of sectional pressure theories, the situation described above is particularly simple, for in them Fig 15 for any one thickness gives, at the same time, the diagram for any lesser thickness, as the penetration curves for the second are already represented in the diagram, i.e. in this case, the direction of the penetration curve at any point is independent of its position with respect to the face of the target, as it is a function only of the velocity. Fig 15 can then be obtained directly from Fig 14; the rupture line in the former is gotten from the latter by following the limiting penetration curve from R to the point at which it is tangent to the scabbing curve, and thence following the scabbing curve to O' . The accompanying Fig 16 shows the simpler possibilities which may occur, depending on the shape of the penetration curve relative to the scabbing curve, following the more explicit treatment of L. Montigny*. Here p and s are the penetration and scabbing curves, respectively.

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*1936.3, pp. 387-393.

8...Penetration in Earth and Concrete:

The foregoing review of the literature on theories and data on the penetration process has served to display the various possibilities which are already available, at the outset, to investigators new in the field. We may here sum up tentative conclusions concerning penetration in these substances.

8a...Dependence on Weight and Size of Bomb:

The observations on penetration recorded above and in the Notes thereto seem to warrant the expectation that, on comparing the penetration of bombs of similar shape, it will be found to be directly proportional to the sectional pressure P of the bomb. In testing this, it is of great advantage to choose as large a range of caliber as possible; thus, the observations upon which de Giorgi's empirical formulae are based extended over a caliber range all the way from rifle bullets to 24 cm mortar shells. Whether the form of the bomb can adequately be taken into account by the multiplicative form factor $1/i$ seems more questionable; thus in the Poncelet formula, the nose form may affect the value of the shatter strength a differently from the coefficient b of the inertial term. It may be found desirable to test this point;if the suggestions given below concerning the determination of a are followed, it may be possible to determine the effect on a by the methods there described.

Aside from the question of the possible effect of form of the projectile, the conclusion so far is that for similar shapes, the resistance and maximum penetration can be represented by formulae of the type

$$(8.1) \qquad R = A\, f(v)\,, \qquad \bar{x}_1 = P\, F(v_0)\,,$$

where A is the maximum cross-sectional area $\pi(d/2)^2$ of the projectile, P is its sectional density W/A, and F and f are related with each other by eq (1.3).

8b...Dependence on Striking Velocity:

The investigator is next confronted with the problem of determining either the dependence of resistance on velocity, and hence of depth of penetration with the aid of eq (1.3), or with that of determining this latter by direct observation; it should be found advantageous to carry both of these methods as far as possible. On this question of velocity dependence, the classical results on penetration are in no such unanimity as on the dependence on sectional pressure. This is well illustrated by Pl I at the end of the report, where the penetration per unit sectional density of the bomb is plotted according to the formulae of Robins-Euler, Poncelet, Pétry and de Giorgi (for masonry), extrapolated out to striking velocity of 2,700 ft/sec in the cases in which the dependence on velocity is expressed in analytical form, i.e. in all except that of de Giorgi, in which his own extrapolation from 1,280 to 1,580 ft/sec is included. Here the multiplicative parameter depending on the material has, in each case, been chosen so as to be in reasonable agreement with the others somewhere in the range 800-900 ft/sec , and the second Poncelet parameter b/a is that for limestone. It will be observed that the velocity dependence of the empirical de Giorgi formula for masonry agrees very well with the Poncelet semi-empirical formula for limestone over the range in which the former is given, and that the Robins-Euler formula has a much smaller slope than the others, which may be interpreted to mean that it yields a greater increase in penetration per unit increase of striking velocity. Now the work of de Giorgi shows that it is practicable to use small caliber arms in testing the dependence of penetration on sectional pressure; this suggests that the same procedure may be used to advantage here in testing its dependence on striking velocity. Thus, a series of observations on penetration in concrete of rifle or machine-gun bullets, with a velocity range of, say, 500 to 2,700 ft/sec should facilitate the choice between these existing types of curves -- or, in case none of them is found adequate, to set up an alternative formula. Any information available on the depth of penetration of larger caliber guns should, of course, also be examined with respect to this point; if these are found to agree with respect to velocity dependence with those obtained by small caliber work, the latter offers a quick and inexpensive means of investigating the finer details.

It is to be observed that, over a sufficiently limited velocity range, any function of v_0 of the type here involved may, by proper choice of parameters, be made to agree with any other, within some given degree of approximation--say, as in the work of Zaleski*, with a suitable chosen linear function. But such an ad hoc procedure, unaccompanied by correlation with physical properties of the material, is not to be recommended, for even though such a formula might seem to be adequate for a given material within a limited velocity range (as in the case of the linear approximation mentioned on p.18 above to the de Giorgi masonry formula), it could not be applied with any degree of confidence to higher velocities.

8c...Dependence on Physical Properties of Target Material:

The point on which there is more disagreement among the various penetration formulae, is in the correlation of the parameters with physical properties of the medium in question. Thus, the various formulae given for "concrete," plotted in Pl II, disagree violently among themselves. As a case in point, de Giorgi's values agree fairly well with Pétry's, if the latter be multiplied by the factor 0.6, and Milota reports** that the penetrations actually observed in the walls of the Verdun forts can be accounted for by a formula obtained by multiplying Pétry's by 0.35! This does not necessarily mean that the observations are faulty, but it does indicate that observations unaccompanied by adequate correlation with physical properties are of much less value than they might otherwise be. The fact that Bazant attributes a compressive strength of 2,800 lb/in^2 to the concrete involved in de Giorgi's observations, and that Milota

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*See Vieser 1934,2, p. 310.

**1933.1, p.264.

considers the compressive strength of the Verdun concrete to be 5,600 lb/in^2, does not in itself enable one to make due allowance for these factors; are we, for example, to assume that Pétry's concrete had a compressive strength of 1,900 lb/in^2? This question of correlation with properties is obviously one of the greatest importance, for of all the factors here under consideration, the target is the only one which is completely under the control of the defender.

Any theory of resistance to penetration must, necessarily, take due cognizance of the fact that at least part of the work goes into overcoming the cohesive forces inherent in the concrete and will, therefore, be concerned with the correlation of this "shatter strength" with the results of standard laboratory tests. There seems to be a, perhaps natural, tendency on the part of most writers to assume that the compressive strength of the concrete is the most relevant for this purpose, but the only one of the theories here under review in which the compressive strength enters directly is that of Vieser -- and reasons have been given above for believing that this theory is unsound. Gailer, on the other hand, believes that shatter strength is unrelated to compressive strength, that it may be related to some kind of punching strength, and that it should, in any case, be some fraction of the "crushing strength" -- i.e. of the work required to reduce unit volume of concrete to dust.*

On the other hand, two attempts to solve the inverse problem have been found, i.e. that of determining compressive strength by measurements on penetration, by setting up an empirical correlation between the two. In the first of these, reported by B. G. Skramtajew**, the volume of the crater formed by shooting a revolver bullet into concrete is correlated empirically with the compressive strength of the concrete; a detailed account of Skramtajew's results, together with a derivation of the shatter strength they

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*1937.2.

**1935.2.

imply on the basis of the Euler and Poncelet theories is given in Note 7 below. The second attempt is that of K. Gaede*, in which the standard Brinell test for metals is applied to concrete, and an empirical correlation set up between the Brinell hardness coefficient so obtained and the compressive strength; the results are exhibited in Note 8, together with an analysis of the Brinell procedure. Skramtajew criticizes Gaede's procedure on the ground that such a test gives information only concerning the strength of the concrete within a few millimeters of the surface, and that conditions there may not be representative of the mass. Nevertheless, it seems that either of these methods might be developed into a reliable means of testing shatter strength of concrete, as both of them offer a fairly direct means of measuring quantities relevant to the present problem, regardless of correlation with compressive strength. Perhaps Skramtajew's method could be employed with a very small caliber bullet, or with reduced charge, thus reducing the damage to the structure to negligible proportions. It was suggested in Secs 3a and 6 above that the Brinell test may prove of value in the problem of armor penetration, and if further comparison with the data lend support to this view, some modification of the Brinell test might be found profitable in the case of concrete.

The problem of the penetration of shell or bomb fragments should, in principle, be susceptible to treatment along the lines developed above, but because of the irregular shapes of the fragments this problem contains statistical elements which make precise results more difficult of attainment. It is reasonable to assume, however, that the penetration of such a fragment is considerably less than that of a regular projectile of equal weight and striking velocity. Results of observations on the number, weight distribution and penetration of fragments are given in

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*1934.1.

A R P Handbook No 5;* it is to be noted that the velocities there given range up to 5,000 ft/sec.

Another question which has not been touched upon so far in this report is that of the effect of reinforcing on penetration or perforation of concrete. It seems quite hopeless to attempt a theoretical solution of this problem, and therefore only a brief report is offered of what the writer has been able to gather from the literature. The general consensus of opinion seems to be that reinforcement cannot be counted upon to reduce penetration into concrete;** the opinion is frequently expressed that the process takes place too suddenly to allow the bond between the concrete and the reinforcing to take effect. On the other hand, it seems generally agreed that proper reinforcing will be of considerable importance in preventing scabbing, or in reducing the dangers attendant upon the scabbing or loosening by subsequent explosion of large fragments of concrete.***

8d...Summary:

At the present time, the writer is inclined definitely to favor a working hypothesis of the Poncelet type, taking the resistance to be the product of the maximum cross-sectional area of the bomb and a factor of the form $a + bv^2$, the first term of which is a measure of the stress required to overcome cohesion and the second of the inertial resistance of the resulting detritus. Certain it is that these two elements are present, and the first question to be answered is whether they alone will suffice to give an adequate description of the penetration process.

* * * * * * * * * *

*1939.1.

**Bezant 1937.1, p. 166; Heidinger 1937.3, p. 269.

***See Montigny 1936.3, pp. 407-408; Civil Protection 1939.2, pp. 144-145; A R P Handbook No 5, 1939.1, p. 19.

TABLE I

VALUES FOR μ TAKEN FROM THE LITERATURE AND REDUCED TO lb/in^2

Robins-Euler formula (2.7) p.8

$$x_1 = \frac{P}{2\mu g} v_o^2$$

MATERIAL: Class	Description	μ	SOURCE
EARTH	Loosely heaped	700	1
	- - -	1,900	2
	- - -	2,100	3
	Average compact	28-3,200	1
WOOD	Elm (lead ball)	22,000	2
CONCRETE	- - -	11-17,000	3
	Poor or green	14,000	4
	Exceptional quality	21,000	[illegible]
	Reinforced	21-32,000	3
	Ordinary masonry (1:8-1:10)	28,000	5
	2800 lb/sq in compression	29,000	6
	- - -	31,000	4*
	- - -	32,000	7
	4.5 to 5.4 sacks/cu yd**	43,000	5
	Reinforced	35-50,000	4
WROUGHT IRON	- - -	140,000	8
STEEL	- - -	210,000	3

1...Handbook No 5 1939.1 (p.16)
2...Euler 1745.1
3...Peres 1932.1 (p.25)
4...Heidinger 1936.2 (p.457)
5...Heidinger 1937.3 (p.269)
6...Reduced from Skramtajew's data (See also Note 7)
7...Vieser 1934.2 (p.311)(reduced from Zolesky's data)
8...de Giorgi 1912.1 (p.20) Old Krupp Works' formula

*This value reduced from de Giorgi's table.
**Reduced from the original source value of 250-300 kg/m^3

TABLE II

VALUES FOR a, b, and b/a
TAKEN FROM DIDION 1848.1 (pp. 241-44)
IN METRIC UNITS*

Poncelet formula (2.10) p.11

$$x_1 = \frac{P}{2gbi} \ln \left(1 + \frac{b}{a} v_0^2 \right)^{**}$$

MATERIAL: Class	Description	a kg/cm^2	b $\frac{kg\text{-}sec^2}{m^2\text{-}cm^2}$ 10^{-3}x	b/a $\frac{sec^2}{m^2}$ 10^{-6}x
EARTH				
	Sand-gravel	43.5	8.7	200
	Earth-sand-gravel	60.	12.0	200
	Clay-sand-gravel	104.5	3.7	35
	Grassy earthwork	70.	4.2	60
	Sand-clay earthwork	46.1	2.8	60
	Clay soil	34.5	2.8	80
	Damp clay	26.6	2.1	80
	Wet clay	9.17	0.73	80
	Earthwork	30.4	0.61	20
	Wet earthwork	26.5	0.53	20
MASONRY				
	Good stonework	552.	8.3	15
	Medium stonework	440.	6.6	15
	Brickwork	316.	4.7	15
	Limestone (Metz)	1200.	18.0	15
WOODS				
	Oak, Beech, Hornbeam, Ash	208.5	4.2	20
	Elm	160.	3.2	20
	Fir, Birch	116.	2.3	20
	Poplar	109.	2.2	20

*For British units, see Table II-A

**In the formula the value of the form factor i is dimensionless. Some values have been supplied;

For ball	1	Didion 1848.1, p.234
	3/2	Mayewski; v. Wuich 1893.1
For long shell	2/3	v. Wuich 1893.1; Cranz 1925.1

TABLE II-A

VALUES FOR a, b, and b/a
TAKEN FROM DIDION 1848.1 (pp. 241-244)
IN BRITISH UNITS*

Poncelet formula (2.10) p.11

$$x_1 = \frac{P}{2gbi} \ln \left(1 + \frac{b}{a} v_o^2 \right)^{**}$$

MATERIAL		a	b	b/a
Class	Description	lb/in^2	$\frac{lb\text{-}sec^2}{ft^2\text{-}in^2}$ $10^{-3}x$	$\frac{sec^2}{ft^2}$ $10^{-6}x$
EARTH				
	Sand-gravel	620	11.5	18.6
	Earth-sand-gravel	850	15.8	18.6
	Clay-sand-gravel	1490	4.8	3.2
	Grassy earthwork	1000	5.6	5.6
	Sand-clay earthwork	655	3.7	5.6
	Clay soil	490	3.6	7.4
	Damp clay	378	2.8	7.4
	Wet clay	130	0.97	7.4
	Earthwork	432	0.80	1.86
	Wet earthwork	377	0.70	1.86
MASONRY				
	Good stonework	7850	10.9	1.39
	Medium stonework	6250	8.7	1.39
	Brickwork	4490	6.2	1.39
	Limestone (Metz)	17100	23.8	1.39
WOODS				
	Oak, Beech, Hornbeam, Ash	2960	5.5	1.86
	Elm	2260	4.2	1.86
	Fir, Birch	1650	3.1	1.86
	Poplar	1550	2.9	1.86

*For metric units, see Table II

**In the formula the value of the form factor i is dimensionless. Some values have been supplied:

For ball	1	Didion 1848.1, p.234
	3/2	Mayewski; v. Wuich 1893.1
For long shell	2/3	v. Wuich 1893.1; Cranz 1925.1

3...Comparison of Poncelet Coefficient b with Density (p.14):

In order to examine the hypothesis

$$(2.12) \qquad b = \frac{\gamma}{2}\frac{w'}{g} = \frac{\gamma}{2}\rho'$$

that the Poncelet coefficient b is a measure of the inertial resistance offered by the target material, the Piobert-Morin-Didion values of b were compared with values of the specific weights w' of the media. Table III below gives b (in $gm\text{-}sec^2/m\text{-}cm^3$) from Note 2, values of w' (in gm/cm^3) from various sources, and the values of the dimensionless coefficient γ computed from (2.12), where $g = 9.81\ m/sec^2$. The values of w' found in the literature show a very considerable spread (e.g. "Dry clay" is given in Marks' Handbook as w' = 1, "Clay" in the Smithsonian Tables as 1.8-2.61); no very great accuracy can, therefore, be expected in the resulting values of γ . The values of w' for earths and masonry are taken mainly from Marks' Mechanical Engineers' Handbook, and in case a range of values is there given the mean has been chosen; those for woods are the mean of values found in Marks (M), Smithsonian Physical Tables (S), the International Critical Tables (I), and Eshbach's Handbook of Engineering Fundamentals (E). It is hoped that the values of w' actually used by the Metz Committee may later be available for comparison.

TABLE III

VALUES FOR b, w', AND γ AS REFERRED TO ON PAGE 59, NOTE 3, AND TAKEN FROM VARIOUS SOURCES

MATERIAL	b $\frac{\text{gm sec}^2}{\text{m cm}^3}$	w' $\frac{\text{gm}}{\text{cm}^3}$	γ
EARTH			
Sand-gravel	0.087	1.6-1.9	1.1-0.9
Earth-sand-gravel	0.120	1.6-1.9	1.5-1.3
Clay-sand-gravel	0.037	1.6-1.9	0.5-0.4
Grassy earthwork	0.042	1.6 [a]	0.5
Sand-clay	0.028	1.4	0.4
Clay soil	0.028	1.4	0.4
Damp clay	0.021	1.8 [b]	0.2
Wet clay	0.007	1.8 [b]	0.1
Earthwork	0.006	1.0 [c]	0.1
Wet earthwork	0.005	1.0 [c]	0.1
MASONRY			
Good stonework	0.083	2.6 [d]	0.6
Medium stonework	0.066	2.4 [e]	0.55
Brickwork	0.047	1.9	0.5
Limestone	0.180	2.5 [f]	1.4
WOODS			
Oak, Beech, Ash	0.042	0.66	1.3
Elm	0.032	0.60	1.0
Fir, Birch	0.023	0.50	0.9
Poplar	0.022	0.39	1.1

a...Packed

b...Anomalous behavior of clay seems well known

c...Lightly packed

d...Ashlar

e...Rubble

f...Mean of values from Marks, Smithsonian and International Critical Tables

TABLE IV

VALUES FOR PARAMETER κ REDUCED FROM PÉTRY 1910.1 pp. 63-65

Pétry formula (2.13') p. 15

$$x_1 = \kappa P \log_{10} \left(1 + \frac{v_0^2}{215,000} \right)$$

MATERIAL Class	Description	κ ft-in^2/lb
EARTH		
	Sandy soil	5.29
	Soil (vegetation)	6.95
	Clay soil	10.6
MASONRY		
	Good brickwork	2.95
	Good stonework	1.69
STONE		
	Limestone	0.775[1]
CONCRETE		1.15
		0.405[2]
		0.686[3]

1...From Handbook No 5, 1939.1, p. 15.

2...Kleczke, from Milota, 1933.1.

3...Attributed to Bazant by Handbook No 5, 1939.1, p. 15, but the writer cannot find it there. However, Bazant uses de Giorgi's results, and it is true that with the above value of the coefficient the Pétry formula gives values in good agreement with de Giorgi's.

TABLE V

VALUES OF k TAKEN FROM NOBILE DE GIORGI, 1911.I, pp. 1112, 1113, 1122, 1123

de Giorgi formula (2.15) p.17

$$x_1 = \frac{P}{i} k F(v_o)*$$

MATERIAL Class	Description	k (dimensionless)
EARTH[a]		
	Coarse, broken stone, compact	0.3
	" " " , loose	0.45
	Gravel	0.5
	Coarse, clean sand	0.6
	Earth-gravel	0.8
	Compact or rammed earth	0.8
	Clay soil, or dry compact loam	0.8
	Settled earthwork	0.9
	Dry, fine sand	0.9
	Dry, loose loam	0.9
	Dry, loose field soil	1.0
	Loose, dry earth	1.4
	Damp or wet earth	1.5-1.6
	Stratum of hard coal	1.14
MASONRY[b]		
	Good brickwork	1.0
	Well-cured concrete	0.4

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*The form factor i is given by de Giorgi as follows:

earth (p.1113)

artillery shells with nose fuze	1.0
rifle, type 88	1.3
shrapnel shell	1.3

masonry (p.1123)

A.P. or semi- A.P. shell	1.0
ordinary shell large fuze	1.5

a...From p.1112. Values of $F_e(v_o)$ reduced from de Giorgi's table to standard British units appear in Table V-A

b...From p.1122. Values of $F_m(v_o)$ reduced from de Giorgi's table to standard British units appear in Table V-B

TABLE V-A

VALUES OF $F_e(v_o)$ FOR EARTHWORK REDUCED FROM DE GIORGI'S TABLE p. 1112 TO STANDARD BRITISH UNITS (ft-in^2/lb) AND INTERPOLATED FOR v_o (in ft/sec) FOR INTERVALS OF 50 ft/sec

Note: The values of F_e for v_o greater than 1350 ft/sec are obtained from values extrapolated by de Giorgi.

v_o	F_e	v_o	F_e	v_o	F_e
300	0.57	950	5.40	1600	7.02
350	0.75	1000	5.56	1650	7.12
400	0.99	1050	5.71	1700	7.22
450	1.34	1100	5.85	1750	7.32
500	1.83	1150	5.90	1800	7.43
550	2.51	1200	6.12	1850	7.54
600	3.22	1250	6.24	1900	7.64
650	3.72	1300	6.35	1950	7.75
700	4.15	1350	6.48	2000	7.86
750	4.49	1400	6.58	2050	7.96
800	4.79	1450	6.70	2100	8.07
850	5.01	1500	6.80	2150	8.18
900	5.21	1550	6.90	2200	8.29

The extreme range of penetration about the mean, given by the formula, is estimated by de Giorgi, p.1113, to be:

for velocities up to 660 ft/sec - 45%, + 65%,
" " from 660-980 ft/sec - 30%, + 40%,
" " " 980-1300 ft/sec - 25%, + 30%.

TABLE V-B

VALUES OF $F_m(v_o)$ FOR MASONRY REDUCED FROM DE GIORGI'S TABLE p.1122 TO STANDARD BRITISH UNITS (ft-in^2/lb) AND INTERPOLATED FOR v_o (in ft/sec) FOR INTERVALS OF 50 ft/sec

Note: The values of F_m for $v_o < 500$ ft/sec and for $v_o > 1300$ ft/sec are obtained from values extrapolated by de Giorgi.

v_o	F_m	v_o	F_m	v_o	F_m
100	0.07	600	0.70	1100	1.56
150	0.12	650	0.73	1150	1.65
200	0.17	700	0.86	1200	1.75
250	0.22	750	0.94	1250	1.85
300	0.28	800	1.03	1300	1.94
350	0.34	850	1.12	1350	2.04
400	0.40	900	1.20	1400	2.14
450	0.47	950	1.30	1450	2.23
500	0.54	1000	1.39	1500	2.33
550	0.62	1050	1.47	1550	2.43

5a...Scabbing. De Giorgi:

$$(5.4) \qquad s_1^3 = K_g W v^2 .$$

De Giorgi's values of K_g , p. 1123, reduced to standard unit $[ft\text{-}sec^2/lb]$ are as follows:

good brickwork $K_g = 7.4 \times 10^{-7}$,

well-cured concrete $K_g = 3.0 \times 10^{-7}$.

Values of K_h (and K_g recommended by Heidinger 1937.3, p. 267) reduced to the standard unit $ft^2\text{-}sec/lb$ are as follows:

high quality concrete* $K_h = 1.7 \times 10^{-4}$, $K_g = 2.2 \times 10^{-7}$

reinforced concrete $K_h = 1.0 \times 10^{-4}$, $K_g = 1.5 \times 10^{-7}$.

No values have been computed or observed directly for K_r in the formula eq (5.7) set up in the report, for data on stress-strain relation from which to compute U , eq (5.3), is not available to the writer. But the value

$$K_r = 9 \times 10^{-8} \ sec^2/lb$$

gives a maximum scabbing thickness, eq (5.7) ,which agrees with de Giorgi's for W = 250 lb, v = 700 ft/sec.

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*250-300 kg cement per m^3 concrete, i.e. 4.5-5.4 sacks cement per yd^3.

6...Penetration and Perforation by Conical-Nosed Bullet (p.25):

In order to illustrate the depth effects discussed in Secs 3a and 6, the resistance and velocity curves for the purely fictional case of a conical-nosed bullet piercing a relatively thick armor plate have been computed. The results of this computation are given on the accompanying Plate III, in which

A: The curve marked "Area" shows the rise of the area of impression $A = A(x, \epsilon)$ from 0 at impact to its maximum value, and its subsequent drop to 0, in accordance with the equation

$$(6.1) \qquad A(x,\epsilon) = A(x) - A(x-\epsilon) \; .$$

R: The resistance is shown on the hypothesis (6.2) - (6.5), with γ taken as one, whence

$$(6.5') \qquad R(x,v) = A(x,e) \left[a + \tfrac{1}{2} \rho' v^2 \right] .$$

v: The velocity curve, showing the decrease in v from striking velocity v_0 to its residual value after perforation, is computed from

$$(6.6') \qquad m \ln \frac{1 + \alpha v_0^2}{1 + \alpha v^2} = m'(x) \; .$$

The first two curves show at the same time the characteristic differences between the resistance on the Euler assumption $f(v) = \mu$, const., and the Poncelet assumption (2.8), for the unit of area on the Plate has been so chosen that the total area under the curve A is equal to that under the Poncelet resistance curve R ; the curve A , therefore, shows at the same time the Euler resistance in proper scale, i.e. the resistance which would be encountered by a projectile losing the same amount of energy in perforating the plate as under the Poncelet assumption. The most characteristic feature is the displacement, to be expected, of the maximum resistance to a lower value of x, and a corresponding shifting of the entire curve in the direction of lower penetration depths.

The curves were computed with the assumed values:

Projectile:	Caliber	$d = 0.3$ in,
	conical height	$\Theta d = 2d$,
	over-all length	$2\Theta d = 4d$,
	specific weight	$w' = 0.30$ lb/in^3,
	striking velocity	$v_0 = 2700$ ft/sec.
Target:	Thickness	$c = 0.9$ in,
	specific weight	$w' = 0.25$ lb/in^3,
	Poncelet coefficient	$a = 2 \times 10^5$ lb/in^2.

The energy loss is 2.22×10^4 in-lb, and the value of the corresponding Euler coefficient computed therefrom is $\mu = 3.48 \times 10^5$ lb/in^2, as compared with the Poncelet stress $a = 2 \times 10^5$ lb/in^2; the efficiency n, eq (3.5) of the process is therefore 0.58.

7... Skramtajew's Penetration Test (p. 53):

As mentioned in Sec 3, B. G. Skramtajew 1935.2 correlates the volume V of the crater produced in concrete by a revolver bullet with the compressive strength of the concrete; his results are given in the first two columns of Table VI below. In order to connect these results with the present investigation, the corresponding values of the Euler coefficient have been computed:

$$\mu = E_0/V \tag{3.7'}$$

and the Poncelet coefficient

$$a = \frac{W}{2gV}\,\frac{1}{\alpha}\ln\left[1 + \alpha v_0^2\right], \tag{3.8'}$$

where the Metz value $\alpha = b/\dot{a} = 15 \times 10^{-6}$ sec^2/m^2 has been adopted; the values of μ and a thus computed are given in the last two columns of Table VI. The bullet used by Skramtajew was a nickelled ball weighing 7 gm and having a velocity of 275 m/sec at a distance of 8m.

TABLE VI

COMPARATIVE VALUES OF σ (SKRAMTAJEW) μ (EULER) AND a (PONCELET) FOR VARIOUS VALUES OF V

V cm^3	σ kg/cm^2	μ kg/cm^2	a kg/cm^2
15	50	180	120
5	90	540	360
2.5	130	1080	720
1.3	200	2080	1390*

- -

*Note that the Poncelet a for limestone is 1200 kg/cm^2.

This correlation has been extrapolated by Bazant 1937.1, p. 172, to higher compressive strengths.

In Skramtajew's test, the ratio $n = a/\mu$ of the work required to push the projectile to that actually expended turns out to be 2/3. It should be noticed that this ratio decreases quite rapidly with striking velocity, for on the Poncelet hypothesis n, eq (3.5') for limestone, at striking velocity twice that employed by Skramtajew, would drop to 0.38, and to 0.24 if it were three times as fast. Another way of looking at n, in cases where nose and crater effects are negligible, is obtained by noting that it is, at the same time, the ratio of the actual penetration to the penetration which the projectile would attain if the resistance were the same throughout as that encountered at low velocities

8...Gaede's Application of the Brinell Test:

As the result of extensive tests made at the Technische Hochschule in Hannover, K. Gaede, 1934.1, proposed a method of measuring the compressive strength of concrete in completed structures based upon the Brinell hardness test. In the laboratory tests, the standard 10 mm Brinell ball (or, for weak concrete, a steel ball 20.6 mm diameter) was pressed into the surface of the concrete, and the diameter of the impression measured; the ratio of the latter to the diameter of the ball was then found to have the correlation with the measured compressive strength of the concrete shown in Fig 17. It is not possible, as in the case of Skramtajéw's test, to obtain the corresponding Poncelet shatter strength a , for Gaede does not give the magnitude of the force used to press the ball into the concrete.

Whatever the value of this method may be in determining the compressive strength, it does give a direct measure of the "shatter strength" as used in this report, for the Brinell coefficient may be thought of as a measure of the pressure required to cause penetration at negligible velocities. In the standard Brinell procedure (cf Hayes 1939.1, p. 155) a steel ball 10 mm in diameter is pressed into the surface of the metal under a load of 3000 kg for a period of 30 seconds; the Brinell hardness coefficient H is then defined as the ratio of the load in kg to the area of the impression in mm^2 . But this may be taken as a measure of the pressure at this stage of penetration of the ball into the metal--leaving aside for the time being questions concerning the actual distribution of force over the face of the impression.

Plate I. Velocity Dependence of Various Formulae: (Adjusted Parameter Values)

The plate illustrates the theoretical penetration depth in concrete for different striking velocities. computed from the principal sectional pressure theories by adjusting the strength parameters to force an agreement around v_o = 900 ft/sec. Values adopted in plot are

$i = 1$

Euler: . . . . $\mu = 30{,}000\ \mathrm{lb/in^2}$,

Poncelet: Limestone . . . . . $\frac{b}{a} = 1.4 \times 10^{-6}\ \frac{\mathrm{sec}^2}{\mathrm{ft}^2}$,

$a = 20{,}120\ \mathrm{lb/in^2}$,

Pétry: . . . . $K = .60\ \mathrm{ft\text{-}in^2/lb}$,

de Giorgi (masonry): . . . . . . $k = .34$

In comparing these penetration formulae, adjusted to fit around v_o = 900 ft/sec as above, the following table of conversion factors will be found useful:

TABLE VII

Multiply → by ↘ to get ↗		Euler μ lb/in²	Poncelet a lb/in² *	Pétry $1/K$ lb/ft-in²	de Giorgi (masonry) $1/k$
Euler	μ	1	1.49	1.80×10^4	1.02×10^4
Poncelet	a	0.671	1	1.52×10^{-2}	9.58×10^{-3}
Petry	$1/K$	5.57×10^{-5}	65.9	1	0.559
de G. (m)	$1/k$	9.83×10^{-5}	104.4	1.79	1

*The Poncelet values refer to limestone.

Plate II. Comparison of Various Formulae:
(Original Parameter Value)

The Plate gives the penetration depth per unit sectional pressure predicted by the various formulae, using accepted values of the strength parameters,

$i = 1$

Euler-Peres:* . . . . $\mu = 17{,}000\ \mathrm{lb/in}^2$,

Poncelet: Limestone . . . . $\frac{b}{a} = 1.4 \times 10^{-6}\ \frac{\mathrm{sec}^2}{\mathrm{ft}^2}$,

$a = 17{,}100\ \mathrm{lb/in}^2$,

Pétry: . . . . $K = 1.15\ \mathrm{ft\text{-}in}^2/\mathrm{lb}$,

de Giorgi (masonry): . . . . $k = 0.4$

Plate III. Perforation of Thick Plate by Bullet:

See Note 6 for description.

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*ARP Handbook No. 5 (p. 16) recommends $\mu = 1{,}200\ \mathrm{kg/cm}^2$ for "all except poor quality concrete." Also, it should be noted that, because of its more direct physical significance, the reciprocal of the parameter μ introduced by Peres and used by Handbook No. 5 has been used throughout.

Plate IV. Penetration of Projectiles and Bombs in Earth:

The data on seven penetrations in earth recorded at the Aberdeen Proving Ground, are plotted together with theoretical curves whose strength parameters have been chosen to give reasonable fit. The data used in preparing this plot are given in Table VIII.

TABLE VIII WEIGHT (W), STRIKING VELOCITY (v_o), SECTIONAL PRESSURE (P), PENETRATION DEPTH (x_1), ALONG PATH: RECORDED AT ABERDEEN PROVING GROUND AND USED IN PLOTTING PLATE IV.

	W lb	v_o ft/sec	P lb/in^2	x_1 ft
BOMBS				
T2	600	665	3.40	[illegible]
M30	100	502	1.99	5.5
PROJECTILES				
3" AA M42	12.7	800	1.80	5.7
75 mm Mk I	12.2	530	1.79	3.9
75 mm M48	15	910	2.20	5.5
105 mm M38 A1	32.8	900	2.45	8.6
155 mm T2	95	1075	3.24	12.1
8" Mk I	200	1010	3.98	17.1

TABLE IX — VALUE OF STRENGTH PARAMETER ADOPTED,* AVERAGE DEVIATION FROM MEAN, MAXIMUM DEVIATION FROM MEAN, AND STRENGTH PARAMETERS FROM LITERATURE, AS USED IN PLOTTING PLATE IV.

Formula	Parameter	Av. Dev.	Max. Dev.	Literature Value
Euler	μ = 3220 lb/in^2	34%	60%	2800-3200 cf Note 1
Pétry	$1/K$ = 0.190 lb/ft-in^2 (K = 5.27 ft-in^2/lb)	15%	44%	K = 5.29 cf Note 4
de Giorgi	$1/k$ = 1.39 (k = 0.72)	21%	51%	k = 0.80 cf Note 5
Zaleski	$1/\varsigma$ = 258 lb/in^2-sec	13%	41%	not plotted. straight line passing through origin

Values for Vieser's theory have also been computed, but the strength parameter falls at the extreme minimum of the range given by Vieser; the 600 lb bomb cannot be fitted with others.

- -

*Taken as the computed mean of the parameter value for eight penetrations.

* * * * * * * * *

GLOSSARY OF SYMBOLS USED

Following is a list of symbols used in the report, together with

a...Verbal definition of each symbol,

b...Physical units in system adopted as standard for report, for corresponding Continental units replace in by cm, ft by m, lb by kg,

c...Reference to page where introduced.

a........shatter strength in Poncelet theory; lb/in^2; 11.

b........inertial coefficient in Poncelet theory; $lb\text{-}sec^2/in^2\text{-}ft^2$; 11.

d........caliber of projectile or bomb ; in; 6 .

$d(x)$.....diameter of projectile at distance x from nose; in; 22.

e........thickness of target; ft; 39.

$f(v)$.....velocity-dependent resistance factor; lb/in^2; 3.

f_o.......limit of $f(v)$ for vanishing velocity; lb/in^2; 23.

$f_n(v_o)$...Pétry penetration function ; dimensionless; 15.

$\bar{f}$........penetration coefficient in limiting velocity ; dimensionless; 42.

g........acceleration due to gravity; 32.2 ft/sec^2 ; 3 .

h........ogival height of projectile; ft; 22.

i........form factor; dimensionless; 6.

k_L.......velocity coefficient in Levi-Civita A'; sec/ft; 16.

k........material coefficient in de Giorgi formula; dimensionless; 17.

$\bar{k}$........coefficient in Thompson formula; $lb^{\frac{1}{2}}/in$; 43.

ℓ_1.......maximum penetration measured along path; ft; 29.

m........mass of projectile; $lb\text{-}sec^2/ft$; 40.

m'.......mass of target material displaced; $lb\text{-}sec^2/ft$; 41.

n........efficiency of penetration process; dimensionless; 23.

s........distance of nose from rear face of target; ft; 33.

s_1.......height of maximum scab; ft; 36.

t........time; sec; 3.

t_1.......time to reach maximum depth; sec; 12.

Δt.......time interval in scabbing process; sec; 35.

v........velocity of projectile; ft/sec; 3.

v_o.......striking velocity; ft/sec; 4.

v'.......velocity imparted to detritus; ft/sec; 14.

v_n.......normal component of striking velocity; ft/sec; 30.

v_r.......velocity of projectile after scabbing; ft/sec; 33.

w'.......weight per unit volume of target material; $lb/ft\text{-}in^2$; 14.

x........penetration depth; ft; 3.

x_1.......maximum penetration depth; ft; 4.

Δx......distance projectile moves during scabbing; ft ; 33.

A........maximum cross-sectional area of projectile; in^2; 6.

A'.......effective area on Levi-Civita hypothesis; in^2 ; 16.

A(x).....area cross-section at distance x from nose ; in^2 ; 22.

A(x,e)...amplitude of impression for target slab; in^2 ; 39.

E........kinetic energy of projectile; ft-lb; 8.

E_o.......striking energy of projectile; ft-lb ; 26 .

F(v).....auxiliary penetration function; ft-in^2/lb; 4 .

F_e.......de Giorgi F for earth; ft-in^2/lb ; 18 .

F_m.......de Giorgi F for masonry; ft-in^2/lb ; 18 .

$\overline{F}$........normalized Thompson penetration coefficient ; dimensionless; 43.

G(x).....velocity of projectile at depth x ; ft/sec ; 5.

H........Brinell hardness coefficient; (kg/mm^2); 24.

K_g.......de Giorgi scabbing coefficient; ft-sec^2/lb; 34.

K_h.......Heidinger scabbing coefficient; ft^2-sec/lb; 35.

K_r.......scabbing coefficient proposed in report; sec^2/lb; 36.

P........sectional pressure; lb/in^2 ; 6.

R(x,v)...resistance encountered by projectile; lb; 3.

S........force in rupture process; lb; 33.

T^m.......sectional energy in de Brettes' formula ; (m-tonnes/cm^2); 43.

U........scabbing energy per unit area; lb/ft; 33 .

V(x).....volume of impression at depth x ; $ft\text{-}in^2$; 22.

V(x,e)...volume of impression in target slab; $ft\text{-}in^2$; 39.

W........weight of projectile or bomb; lb; 3.

α ......= a/b, coefficient of v^2 in Poncelet formula; sec^2/ft^2; 41.

β ......angle of conical crater; dimensionless; 28.

γ......drag factor in Poncelet theory; dimensionless; 14.

ϵ ......= 2.718... , basis of natural logarithms; dimensionless; 11.

ζ ......coefficient in Zalesky penetration formula; in^2-sec/lb; 18.

θ ......angle of impact; dimensionless; 29.

θ_r......angle at which projectile ricochets; dimensionless; 29.

κ_1......coefficient in Pétry formula; $ft\text{-}in^2/lb$; 15 .

κ ......coefficient in modified Pétry formula; $ft\text{-}in^2/lb$; 15.

λ ......reduced length in calibers of projectile; dimensionless; 42.

μ ......shatter strength on Euler theory; lb/in^2 ; 7.

ν ......exponent of $\cos \Theta$ in oblique impact; dimensionless; 30.

π ......= 3.1416....

ρ ......mean density (mass) of projectile; $lb\text{-}sec^2/ft^2\text{-}in^2$; 41.

ρ' ......(mass) density of target; $lb\text{-}sec^2/ft^2\text{-}in^2$; 41.

σ ......compressive strength; lb/ft^2 ; 26.

σ_x .....principal stress along x-axis; lb/ft^2 ; 26.

τ ......stress in scabbing process; lb/ft^2 ; 33.

$\varphi(x)$ ...depth-dependent resistance factor; in^2 ; 3.

χ ......slope of plate perforation coefficient; dimensionless; 42.

ω ......material coefficient in Vieser formula; dimensionless; 27.

$\Phi(x)$ ...auxiliary penetration function; $ft\text{-}in^2$; 4.

* * * * * * * *

This Bibliography contains only

a...References from which material was actually used in preparing the report, and

b...References to reports on relevant fields.

As such, it represents but a fraction of the references consulted. For the convenience of any one wishing to consult some of the less readily available items, each reference actually consulted is followed by the Union Catalogue designation of the Library from which the item was obtained; in case a copy (photostat, reprint) is in the hands of the Committee, the library designation is followed by an asterisk -- thus a reference followed by "NjP*" is one obtained from the Princeton University Library, of which the Committee now possesses a photostat.

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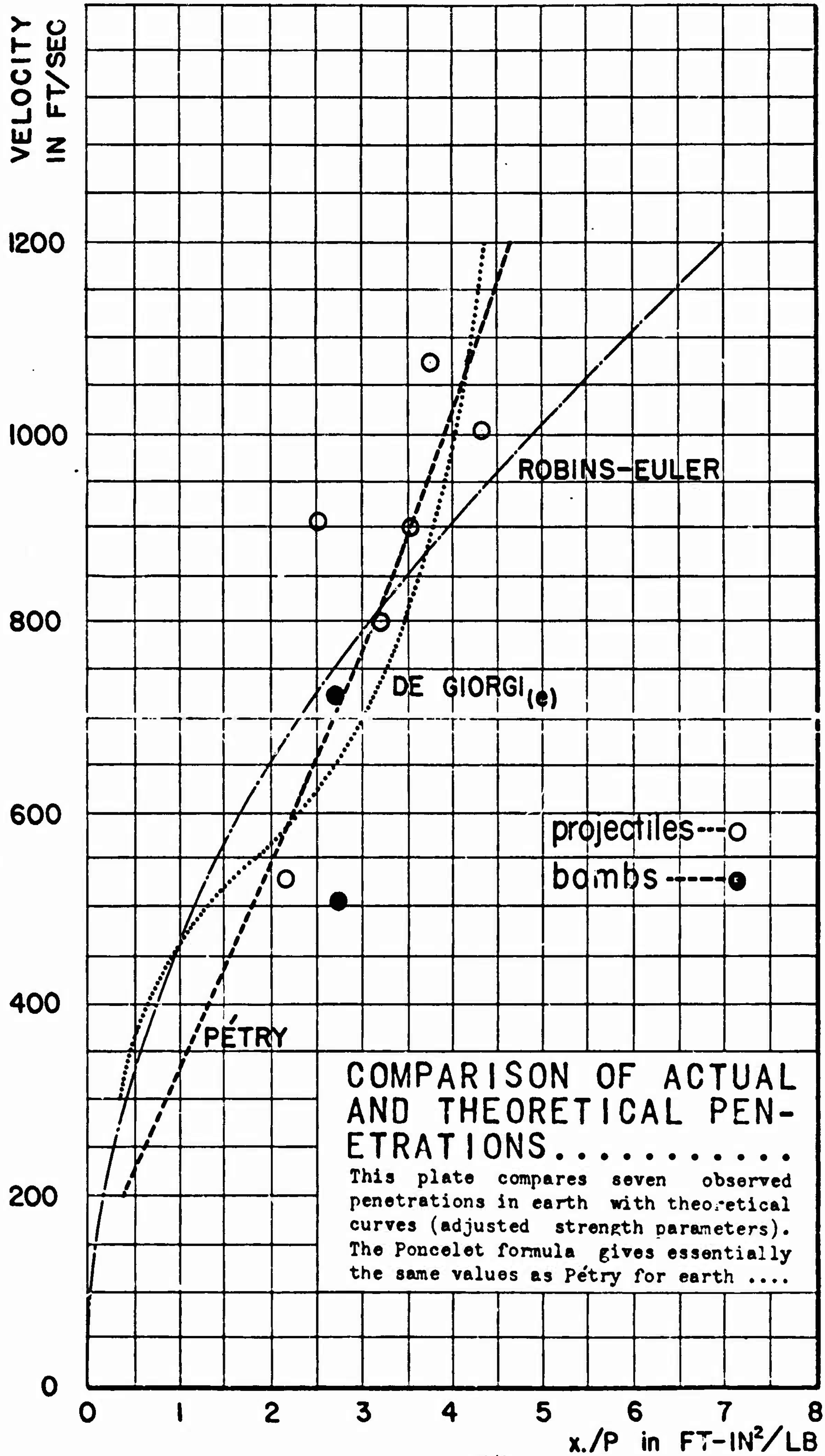


COMPARISON OF ACTUAL AND THEORETICAL PENETRATIONS...........

This plate compares seven observed penetrations in earth with theoretical curves (adjusted strength parameters). The Poncelet formula gives essentially the same values as Pétry for earth ....

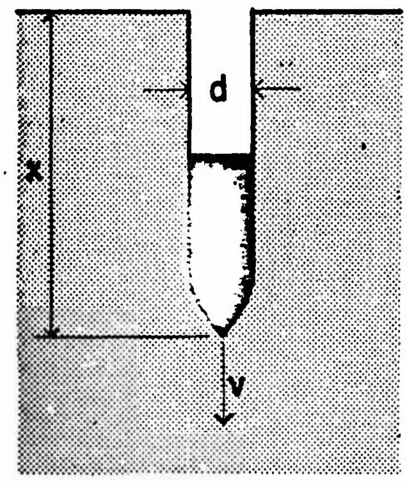


FIG 1

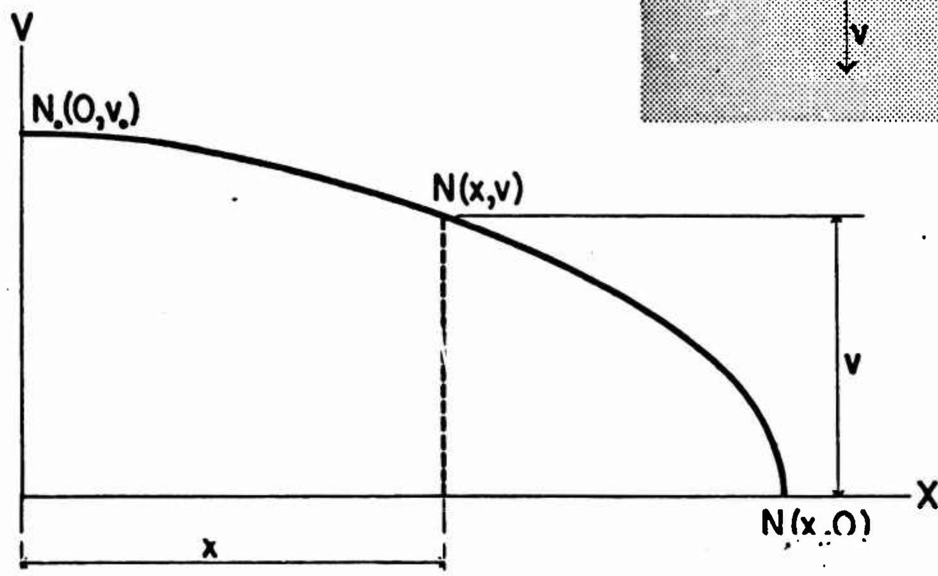


FIG 2

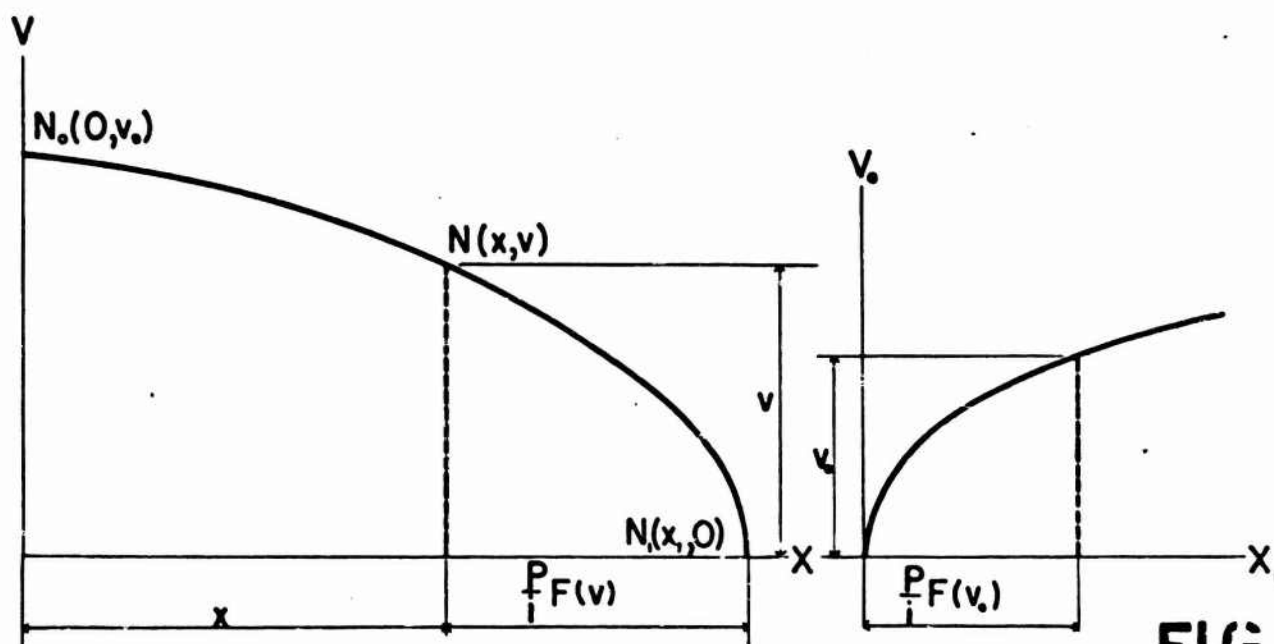


FIG 3

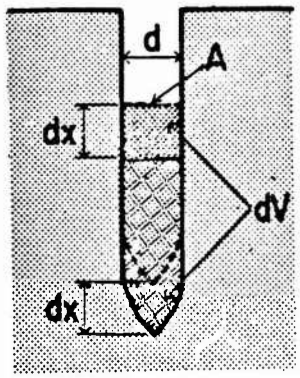


FIG 4

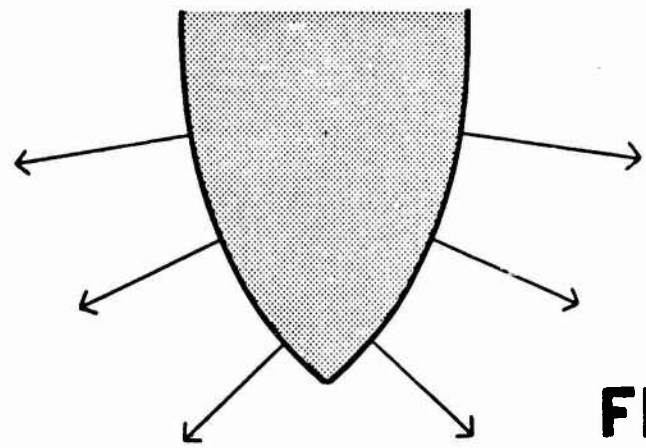


FIG 5

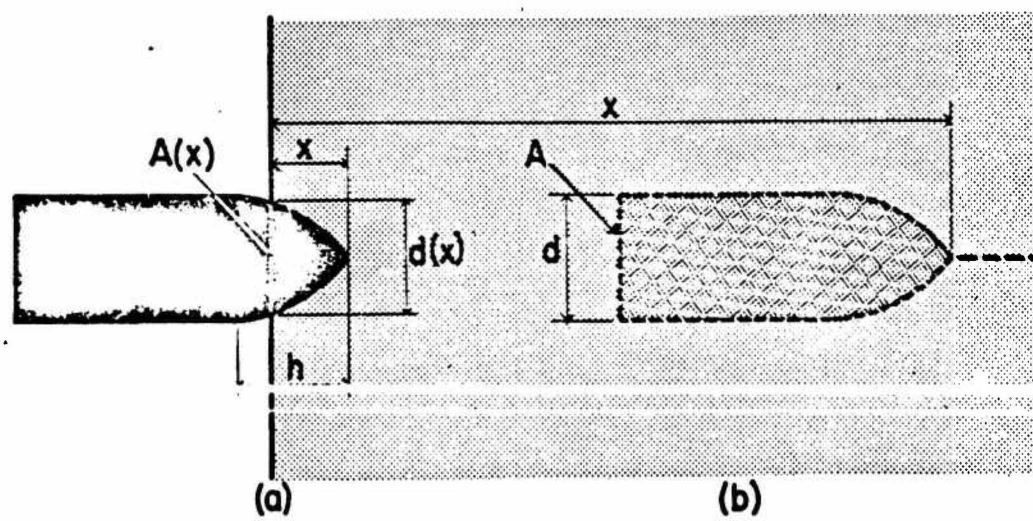


FIG 6

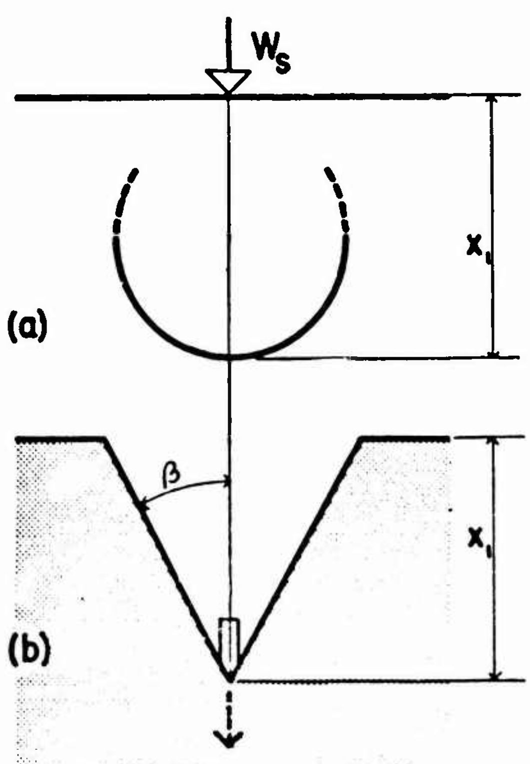


FIG 7

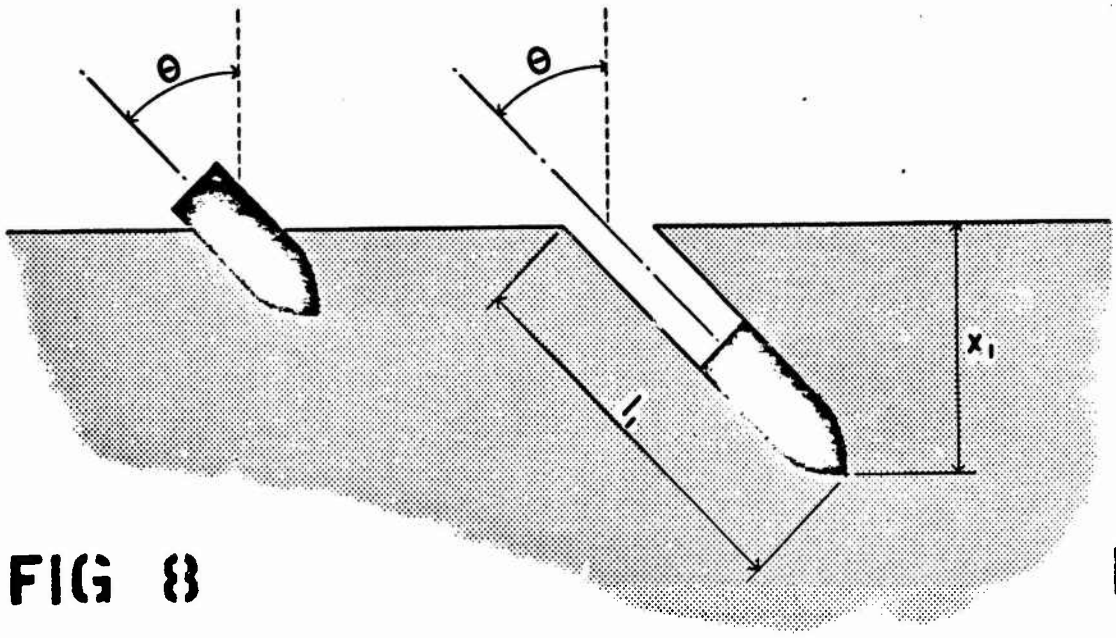
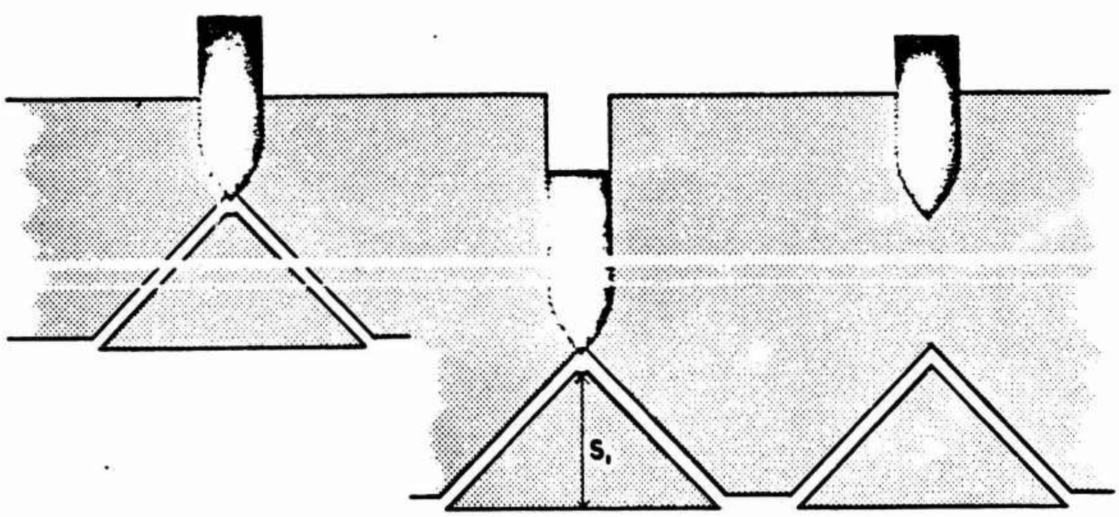


FIG 8

FIG 9



(a) (b) (c)

FIG 10

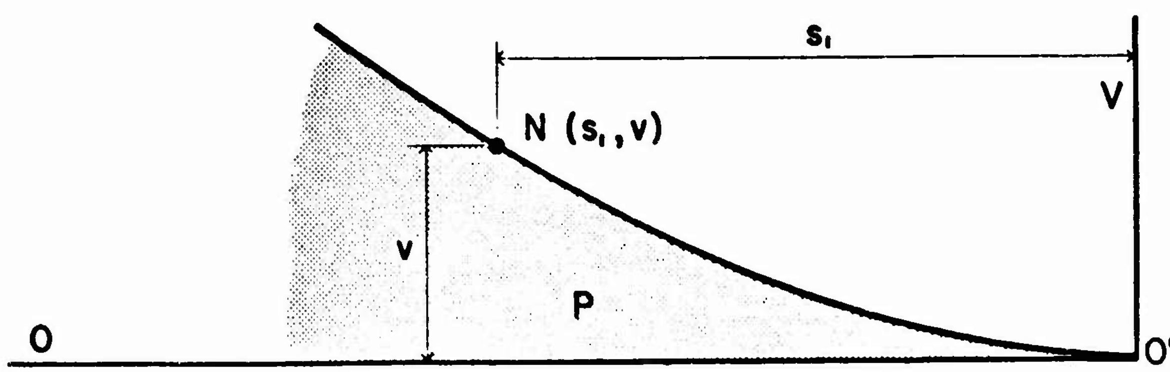


FIG 11

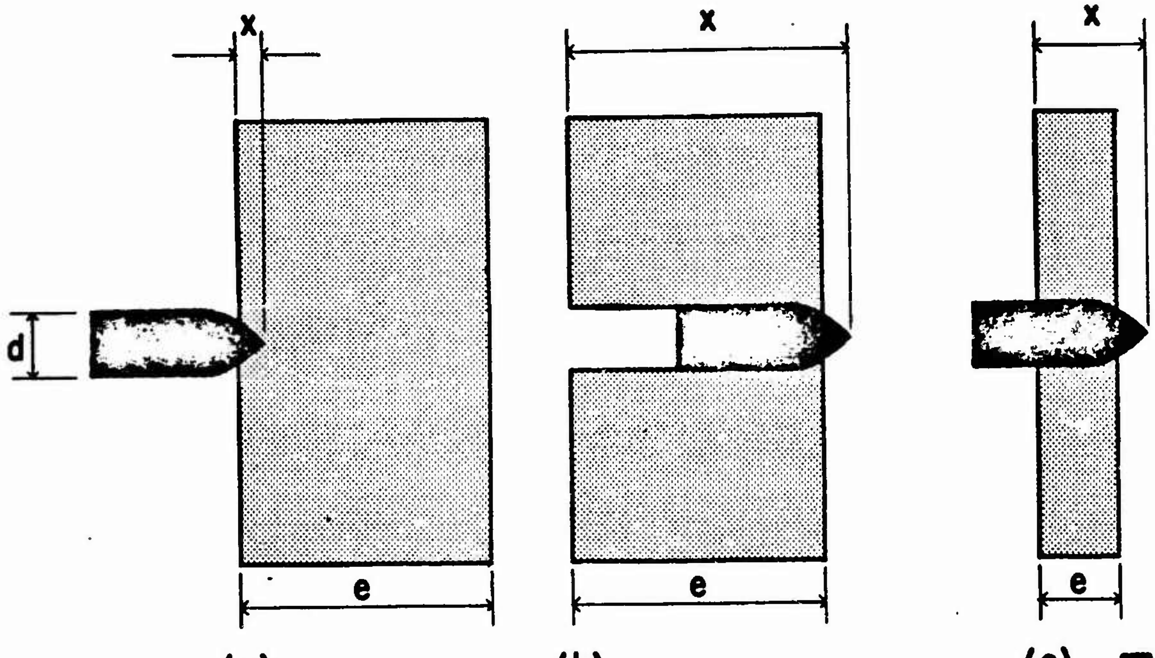

x
x
x
d
e
e
e
(a)
(b)
(c)

FIG 12

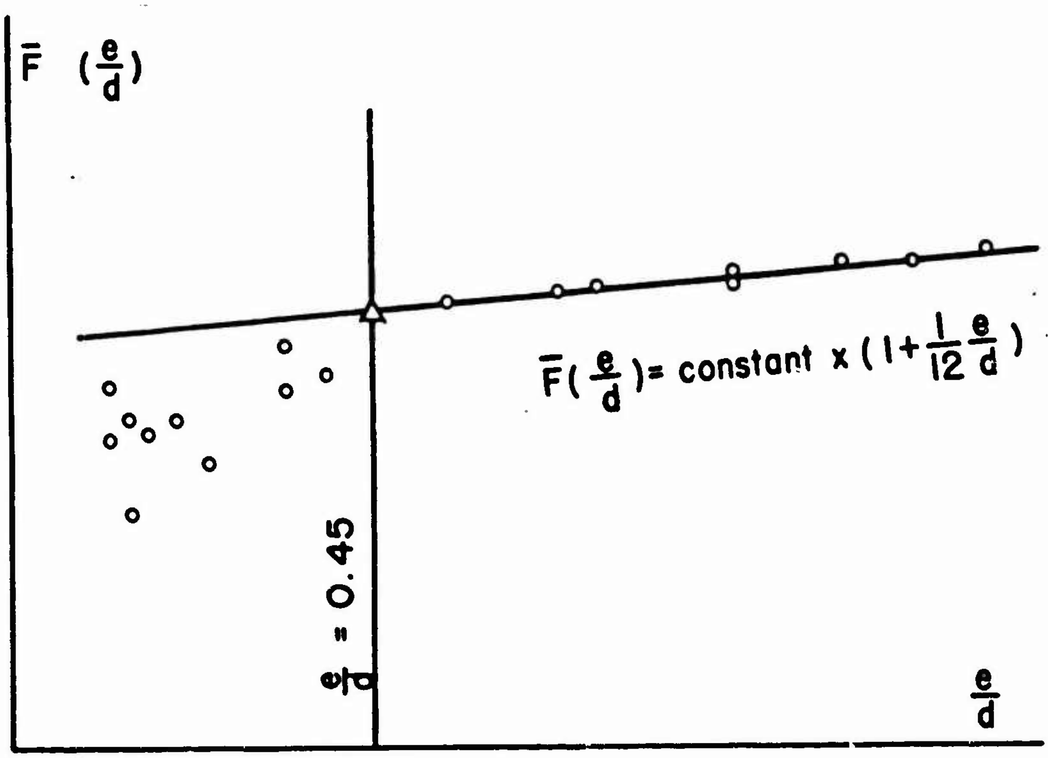
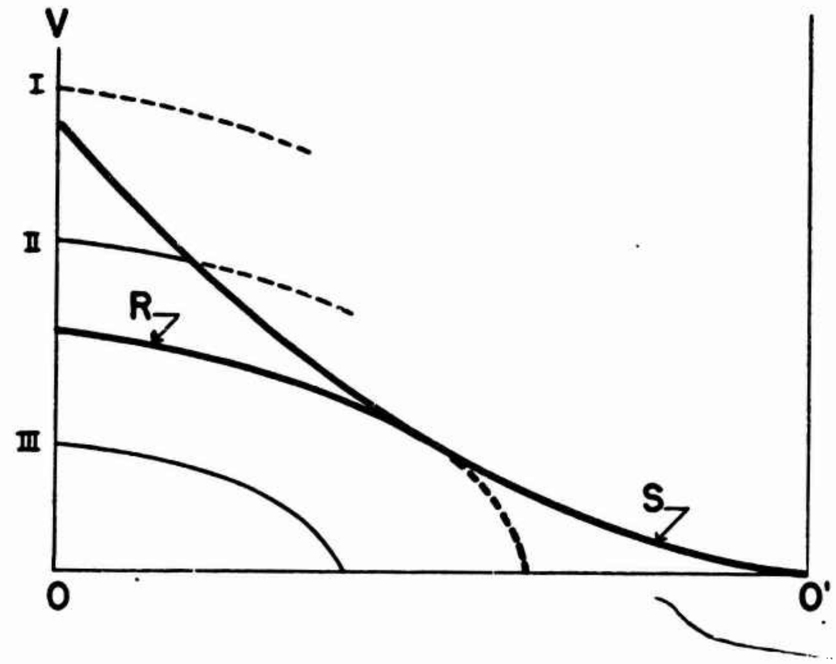
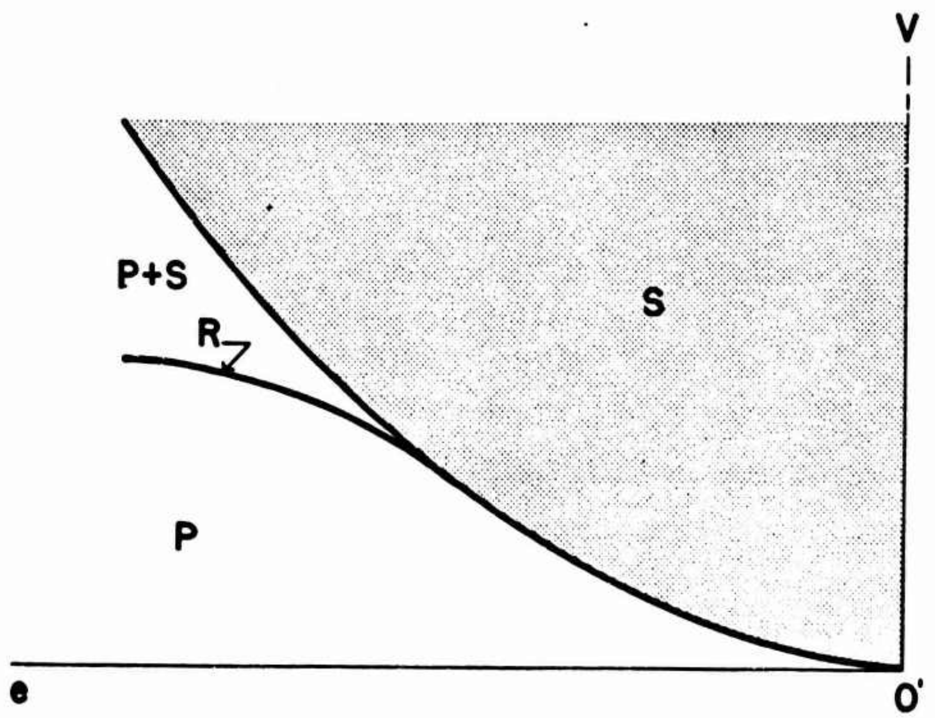

$\bar{F}\ (\frac{e}{d})$
$\bar{F}(\frac{e}{d})$ = constant x $(1+\frac{1}{12}\frac{e}{d})$
$\frac{e}{d}$ = 0.45
$\frac{e}{d}$

FIG 13



V
I
II
R
III
S
O
O'



V
P+S
S
R
P
e
O'

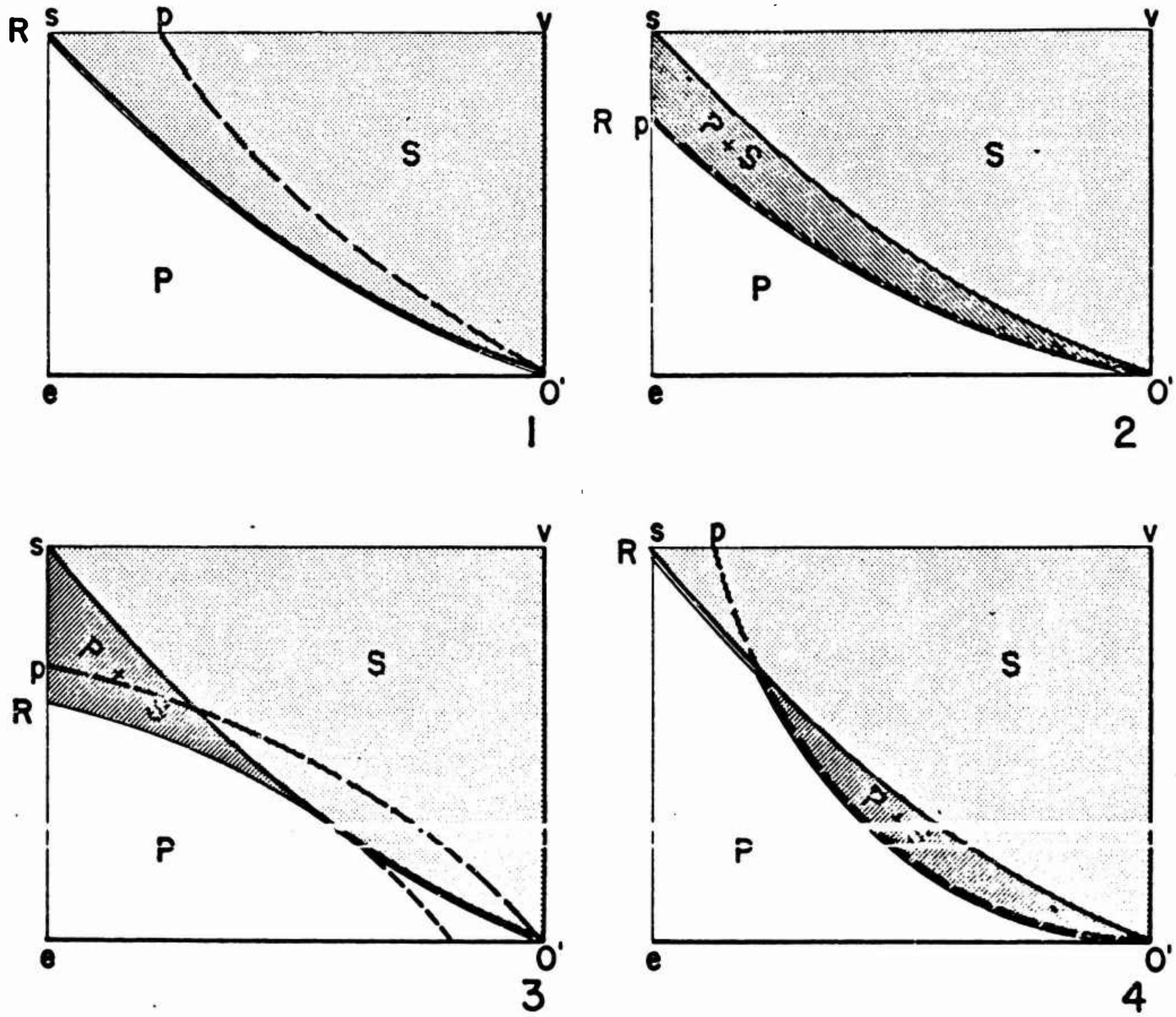

R
s
p
v
S
P
e
O'
1
R
p
P+S
S
P
e
O'
2
s
p
R
P+S
S
P
v
e
O'
3
R
s
p
v
S
P
e
O'
4

FIG 1(

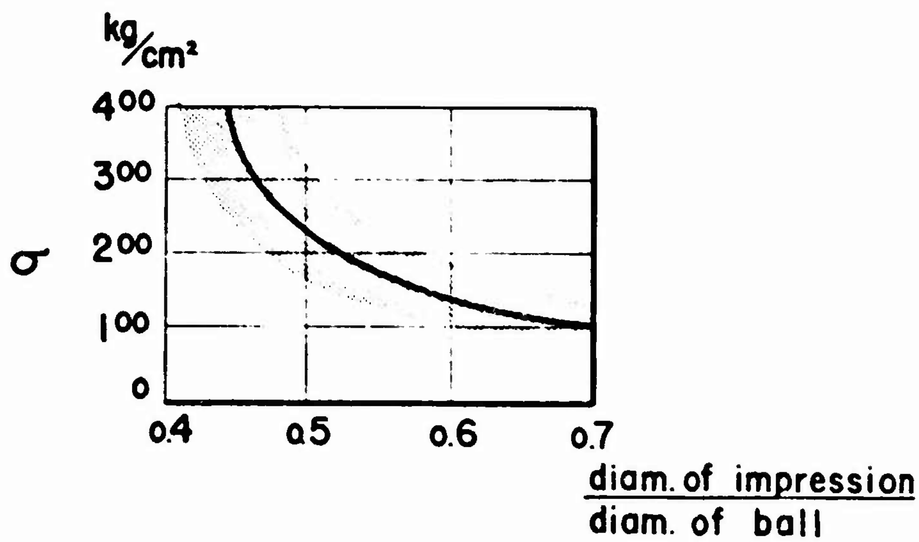
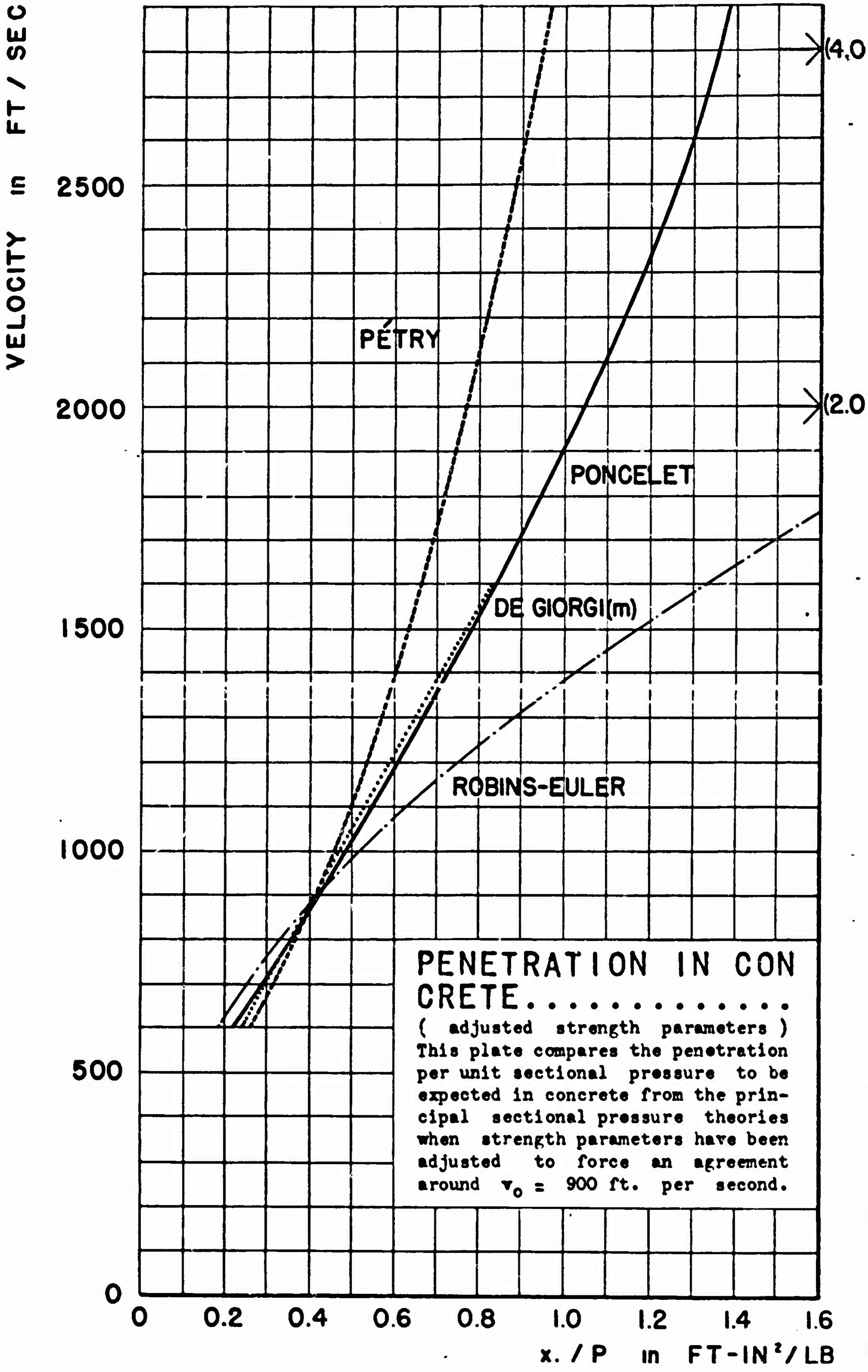

kg/cm²
400
300
200
100
0
σ
0.4
0.5
0.6
0.7
diam. of impression
diam. of ball

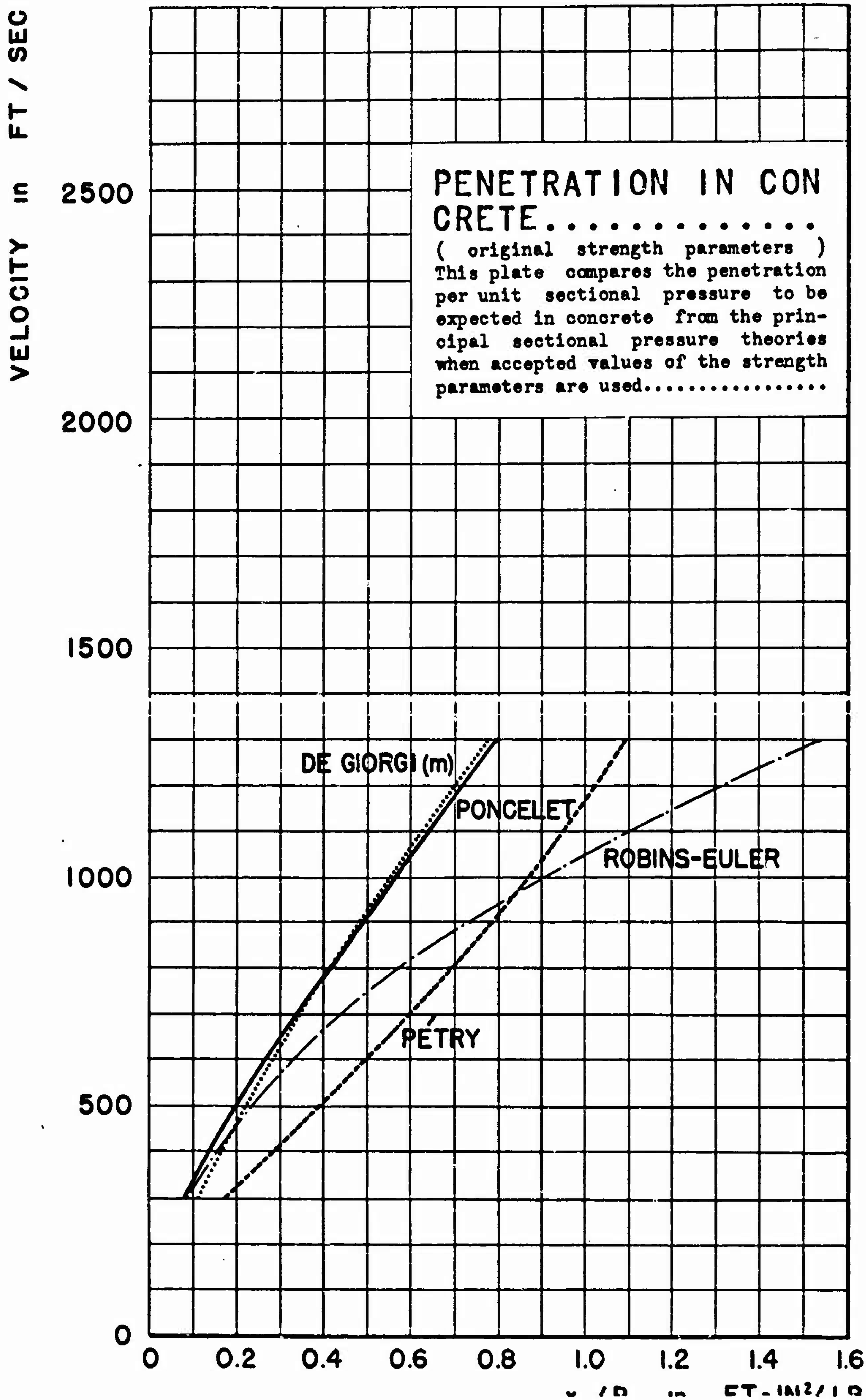
FIG 1'

PLATE i



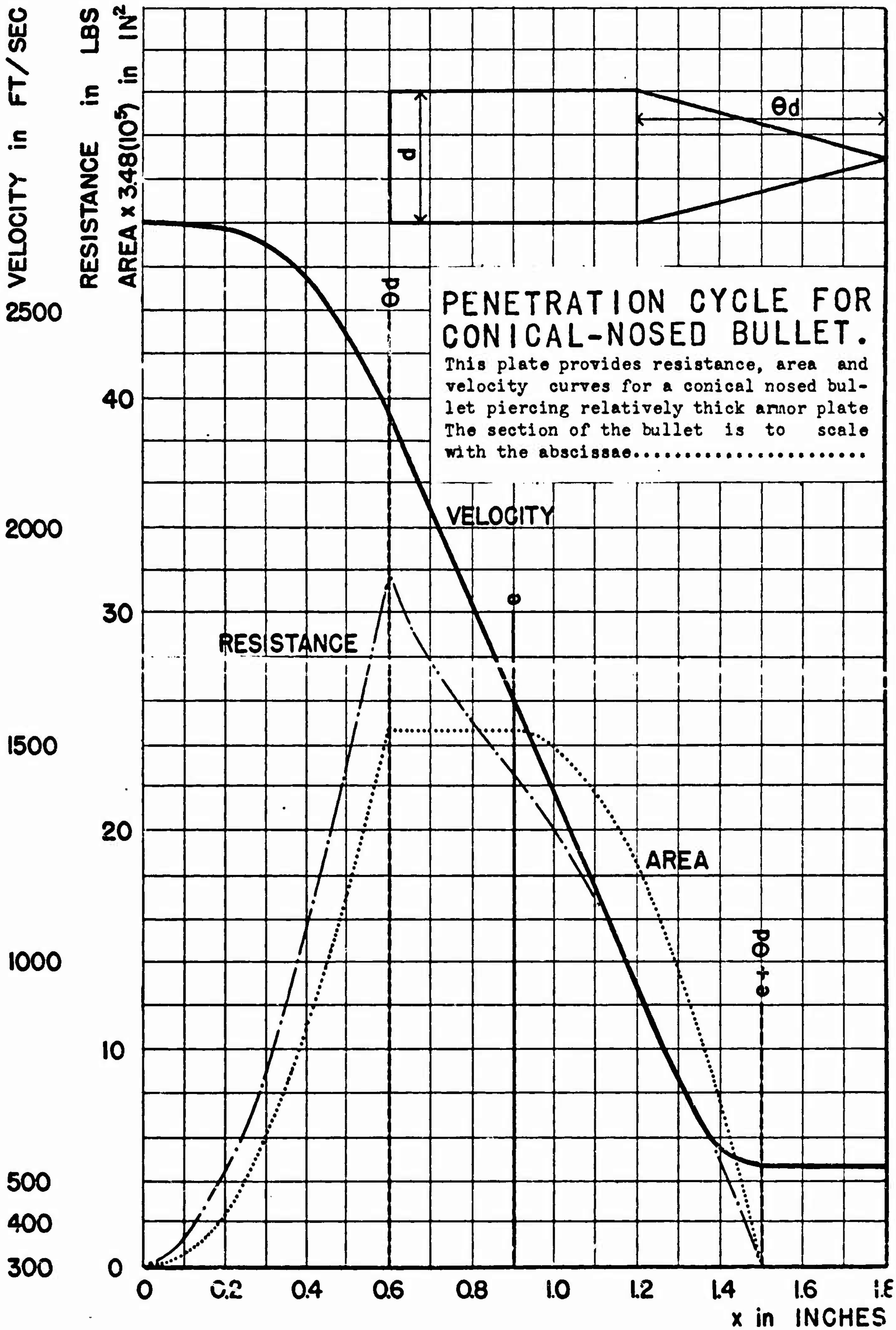
PENETRATION IN CONCRETE...............

(adjusted strength parameters)
This plate compares the penetration per unit sectional pressure to be expected in concrete from the principal sectional pressure theories when strength parameters have been adjusted to force an agreement around v_o = 900 ft. per second.



PENETRATION IN CONCRETE..............
(original strength parameters)
This plate compares the penetration per unit sectional pressure to be expected in concrete from the principal sectional pressure theories when accepted values of the strength parameters are used................
VELOCITY in FT / SEC
2500
2000
1500
1000
500
0
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
DE GIORGI (m)
PONCELET
ROBINS-EULER
PETRY

PLATE 11



VELOCITY in FT/SEC
RESISTANCE in LBS
AREA x 3.48(10⁵) in IN²
PENETRATION CYCLE FOR CONICAL-NOSED BULLET.
This plate provides resistance, area and velocity curves for a conical nosed bullet piercing relatively thick armor plate The section of the bullet is to scale with the abscissae......................
θd
d
e
e+θd
VELOCITY
RESISTANCE
AREA
2500
2000
1500
1000
500
400
300
40
30
20
10
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
x in INCHES